



Which One Doesn't Belong?

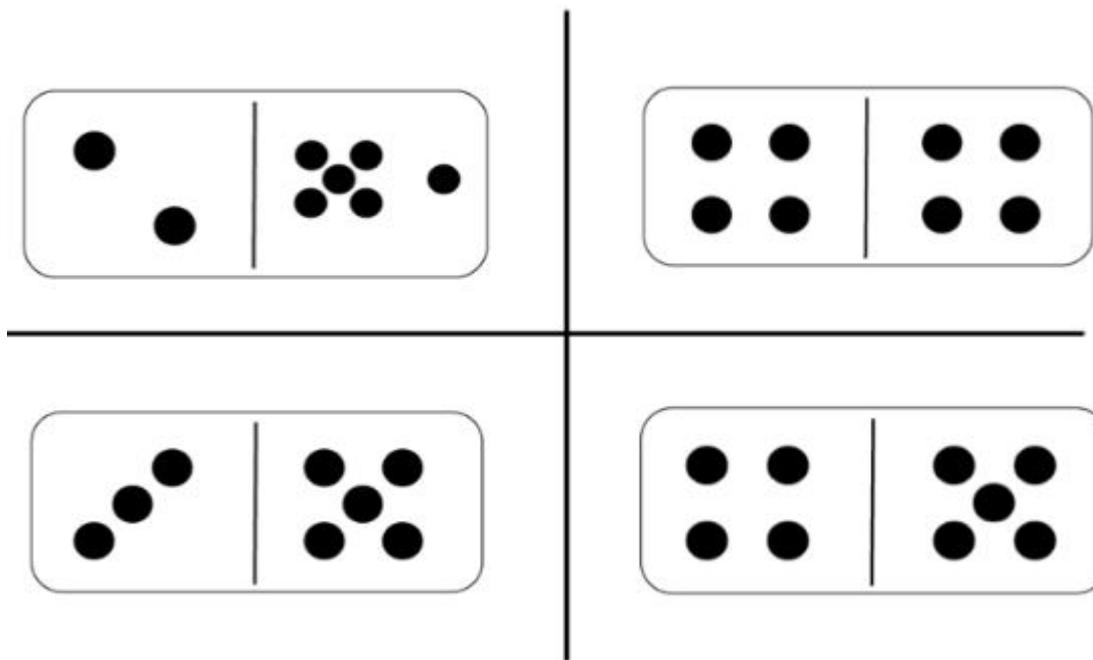
Find a reason why each one does not belong.

2×8 	$8 + 8$
4×4 	8×2



Which One Doesn't Belong?

Find a reason why each one does not belong.





Which One Doesn't Belong?

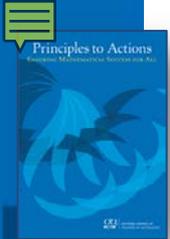
Find a reason why each one does not belong.

$8 + 6$	$7 + 7$
$8 + 7$	$5 + 9$

Which One Doesn't Belong?

Find a reason why each one does not belong.





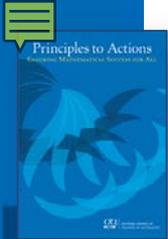
Principles to Actions

Effective Mathematics Teaching Practices

The Case of Katherine Casey and the Multiplication Strings Task Grade 4

This module was developed by DeAnn Huinker, University of Wisconsin-Milwaukee; Victoria Bill, University of Pittsburgh Institute for Learning; and Amy Hillen, Kennesaw State University. Video courtesy of New York City Public Schools and the University of Pittsburgh Institute for Learning.

These materials are part of the *Principles to Actions Professional Learning Toolkit: Teaching and Learning* created by the NCTM project team that includes: Margaret Smith (chair), Victoria Bill (co-chair), Melissa Boston, Fredrick Dillon, Amy Hillen, DeAnn Huinker, Stephen Miller, Lynn Raith, and Michael Steele.

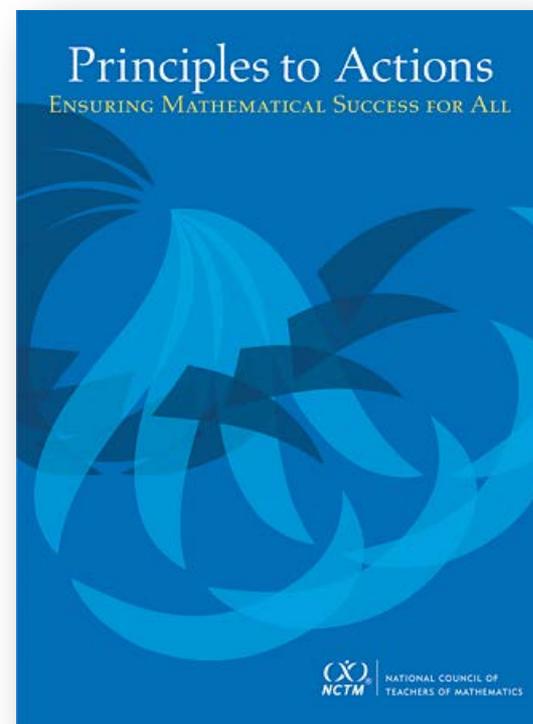


Overview of the Session

- Overview of the Eight *Effective Mathematics Teaching Practices*.
- Watch a video clip of a fourth grade class engaged in a whole class discussion of a series of related multiplication equations.
- Discuss what the teacher does to support the students' learning of mathematics.
- Relate teacher actions in the video to the *Effective Mathematics Teaching Practices*.

Principles to Actions: Ensuring Mathematical Success for All

The primary purpose of *Principles to Actions* is to fill the gap between the adoption of rigorous standards and the enactment of practices, policies, programs, and actions required for successful implementation of those standards.





Guiding Principles for School Mathematics

Teaching and Learning

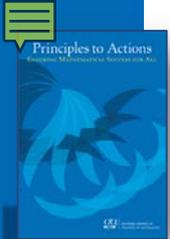
Access and Equity

Curriculum

Tools and Technology

Assessment

Professionalism



Teaching and Learning Principle

An excellent mathematics program requires effective teaching that engages students in meaningful learning through individual and collaborative experiences that promote their ability to make sense of mathematical ideas and reason mathematically.

National Council of Teachers of Mathematics. (2014). *Principles to actions: Ensuring mathematical success for all*. Reston, VA: Author. (p. 7)



At The Wall

Teaching Practices

- **Implement tasks that promote reasoning and problem solving.**
- **Build procedural fluency from conceptual understanding.**
- **Use and connect mathematical representations.**
- **Elicit and use evidence of student thinking.**



Ms. Casey's Fourth Grade Classroom "Multiplication Strings Task"

Multiplication Strings Task (Teacher Version)

1. Solve this set of multiplication equations. Each time you solve a new equation, try to use the previous equation to help you solve it.

$$8 \times 4 = \underline{\quad}$$

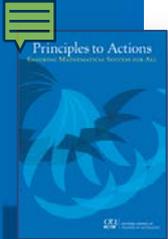
$$8 \times 8 = \underline{\quad}$$

$$8 \times 16 = \underline{\quad}$$

$$8 \times 32 = \underline{\quad}$$

$$8 \times 64 = \underline{\quad}$$

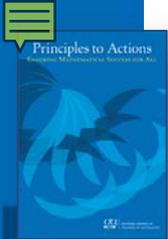
2. Reflect on the string of multiplication equations. What patterns do you notice? Why do these patterns occur?
3. To solve 8×8 , Nicholas, a third grade student, explained that he knew that 8 times 4 would be 32 so he just added 32 and 32 to get the answer. Show how you, as the teacher, might represent his reasoning with an area model. How does his strategy utilize properties of the operations?



Ms. Casey's Mathematics Learning Goals

Students will understand that:

- Multiplication can be represented by the area of a rectangle because tiling the figure would show rows with an equal number or columns with equal number of square units.
- Decomposing and recomposing groups, based on properties of the operations, makes it possible to flexibly and fluently make use of known multiplication combinations to solve unknown multiplication equations.
- Systematically analyzing what happens to the product when one factor is changed reveals patterns and regularity in repeated reasoning that can lead to flexible and fluent multiplication strategies.



Connections to VA-SOL Grade 3 Mathematics Content Standards

Computation and Estimation

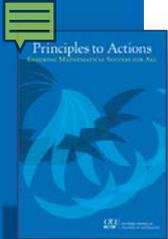
Focus: Computation and Fraction Operations

- 3.6 **The student will represent multiplication and division, using area, set, and number line models, and create and solve problems that involve multiplication of two whole numbers, one factor 99 or less and the second factor 5 or less.**

Patterns, Functions, and Algebra

Focus: Patterns and Property Concepts

- 3.20 The student will:
- a) **investigate the identity and the commutative properties** for addition and multiplication; and
 - b) **identify examples of the identity and commutative properties** for addition and multiplication.

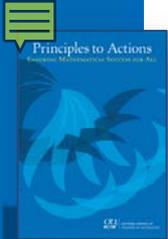


Connections to VA-SOL Grade 4 Mathematics Content Standards

Patterns, Functions, and Algebra

Focus: Geometric Patterns, Equality, and Properties

- 4.16 The student will
- a) recognize and demonstrate the meaning of equality in an equation; and
 - b) investigate and describe the **associative property** for addition and multiplication.



Connections to the VA-SOL Mathematics Goals

1. Mathematical Problem Solving
 2. Mathematical Communication
 3. Mathematical Reasoning
 4. Mathematical Connections
 5. Mathematical Representations
- Discuss how these Virginia Mathematics Goals are related to the tasks.

Classroom Context for the Video Segment

Teacher: Katherine Casey

Grade: 4

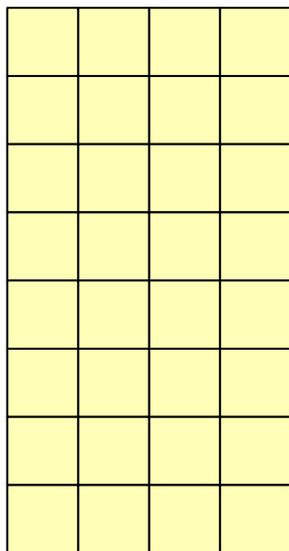
School: PS 116

District: New York Community School District 2

Month: November

Number of students in the classroom: 27

The class has been creating all of the possible rectangular arrays for a given number of square units. In this video, the teacher poses a sequence of equations for the students to solve. Students are challenged to use one equation to solve another equation. The students share their strategies and discuss “what they notice” about the multiplication equations and the products. The teacher uses an area model to represent the students’ reasoning.



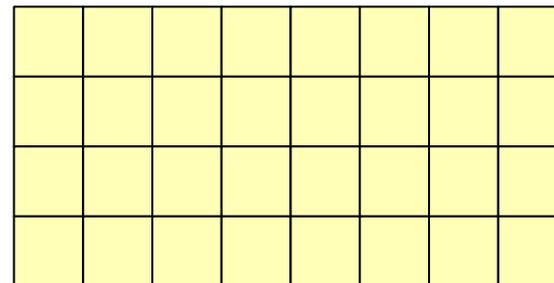
8×4

8 rows with 4 in each row

4 columns with 8 in each column

rotate

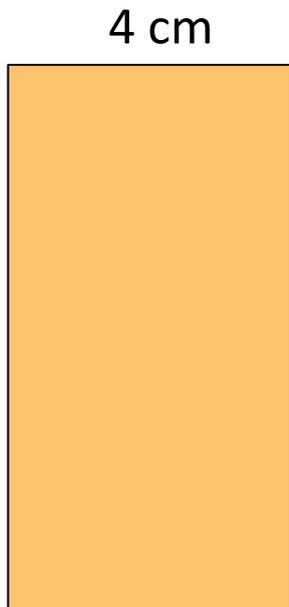
Rectangular Arrays



4×8

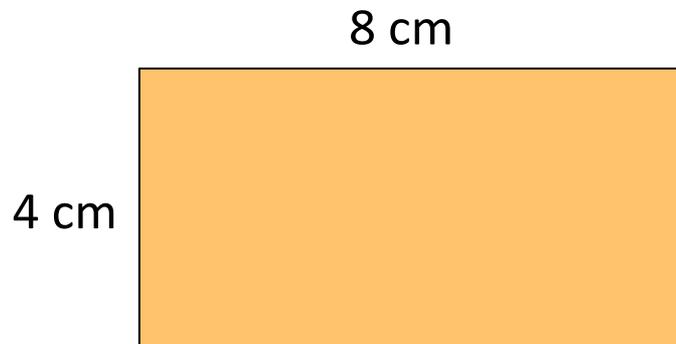
4 rows with 8 in each row

8 columns with 4 in each column



rotate

Area Models



Multiplication Strings: Board Work

4, 8, 12, 16, 20, 24, 28, 32

$$8 + 8 = 16$$

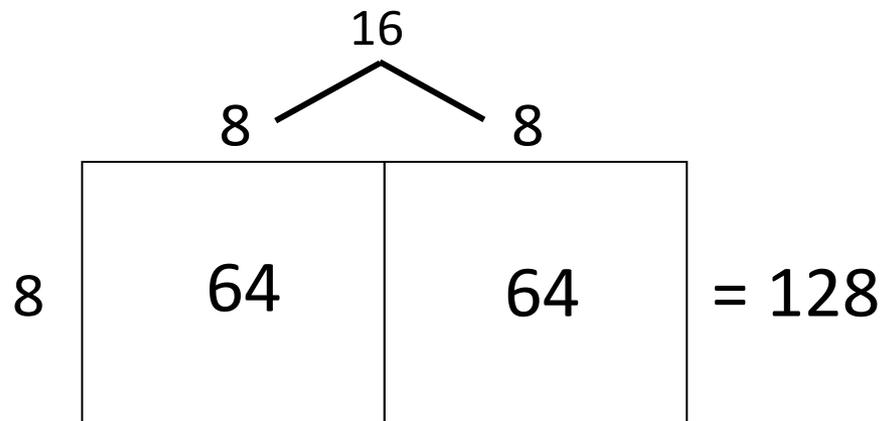
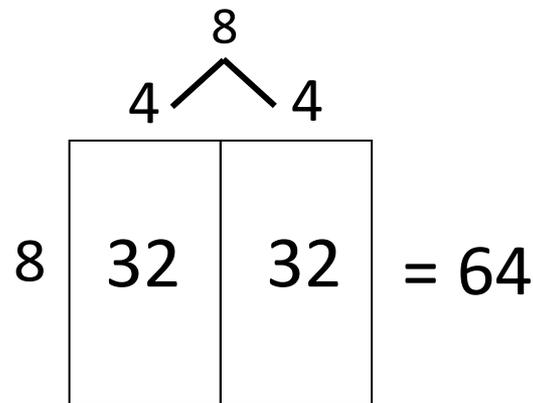
$$16 + 16 = 32$$

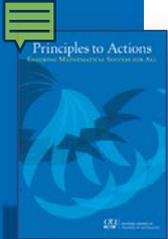
$$8 \times 4 = 32$$
$$\left. \begin{array}{l} 8 \times 4 = 32 \\ \end{array} \right) \times 2$$

$$8 \times 8 = 64$$
$$\left. \begin{array}{l} 8 \times 8 = 64 \\ \end{array} \right) \times 2$$

$$8 \times 16 = 128$$
$$\left. \begin{array}{l} 8 \times 16 = 128 \\ \end{array} \right) \times 2$$

$$8 \times 32 = 256$$





Lens for Watching the Video

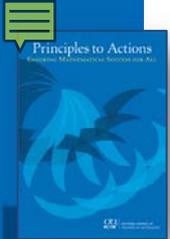
As you watch the video

- Identify the mathematical insights that surfaced for students, and
- Make note of what the teacher does to support student learning of mathematics.



Return to the Wall

Mathematics Teaching Practices	Questions
Tasks	In what ways did the implementation of the tasks allow for multiple entry points and engage students in reasoning and problem solving?
Representations	How did Ms. Casey support students making connections between different representations
Discourse	How was the whole classroom discourse structured to advance student learning?
Questions	In what ways did Ms. Casey's questioning assess and advanced student learning?
Fluency	In what ways did this lesson develop a foundation for conceptual understanding for building towards procedural fluency?
Struggle	At which point(s) in the lesson might Ms. Casey have consciously restrained herself from "taking over" the thinking of students?
Evidence	Identify specific places during the lesson in which Ms. Casey elicited evidence of student learning and then made an instructional decision based on the evidence.



Effective Mathematics Teaching Practices

1. Establish mathematics **goals** to focus learning.
2. Implement **tasks** that promote reasoning and problem solving.
3. Use and connect mathematical **representations**.
4. Facilitate meaningful mathematical **discourse**.
5. Pose purposeful **questions**.
6. Build procedural **fluency** from conceptual **understanding**.
7. Support **productive struggle** in learning mathematics.
8. Elicit and use **evidence** of student thinking.

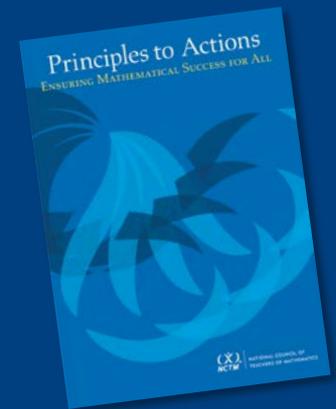


Watch the Video

“Multiplication Strings”



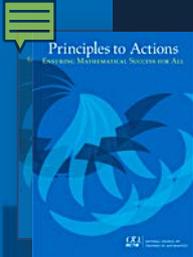
A Look at the Effective Mathematics Teaching Practices



Establish mathematics goals to focus learning.

Formulating clear, explicit learning goals sets the stage for everything else.

(Hiebert, Morris, Berk, & Janssen, 2007, p. 57)



Establish mathematics goals to focus learning

Learning Goals should:

- Clearly state what it is students are to learn and understand about mathematics as the result of instruction.
- Be situated within learning progressions.
- Frame the decisions that teachers make during a lesson.

Daro, Mosher, & Corcoran, 2011; Hattie, 2009; Hiebert, Morris, Berk, & Jensen., 2007; William, 2011



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Implement tasks that promote reasoning and problem solving.

Student learning is greatest in classrooms where the tasks consistently encourage high-level student thinking and reasoning and least in classrooms where the tasks are routinely procedural in nature.

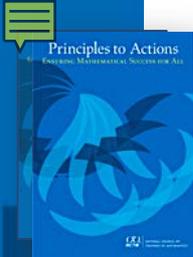
(Boaler & Staples, 2008; Stein & Lane, 1996)



Implement tasks that promote reasoning and problem solving

Mathematical tasks should:

- Allow students to explore mathematical ideas or use procedures in ways that are connected to understanding concepts.
- Build on students' current understanding and experiences.
- Have multiple entry points.
- Allow for varied solution strategies.



Questions

In what ways did the implementation of the task allow for multiple entry points and engage students in reasoning and problem solving?

Use and connect mathematical representations.

Because of the abstract nature of mathematics, people have access to mathematical ideas only through the representations of those ideas.

(National Research Council, 2001, p. 94)



Use and connect mathematical representations

Different Representations should:

- Be introduced, discussed, and connected.
- Be used to focus students' attention on the structure of mathematical ideas by examining essential features.
- Support students' ability to justify and explain their reasoning.

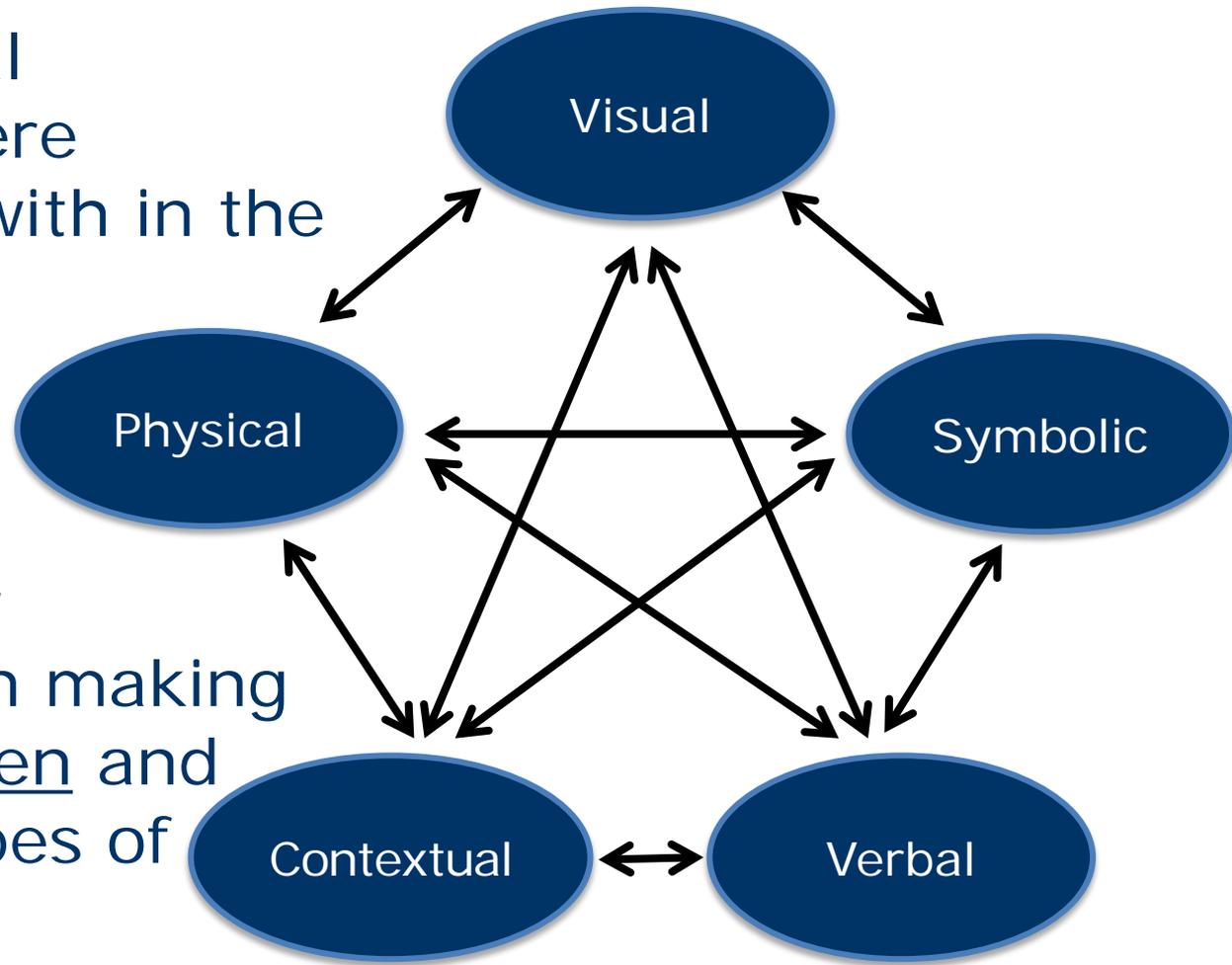
Lesh, Post, & Behr, 1987; Marshall, Superfine, & Canty, 2010;
Tripathi, 2008; Webb, Boswinkel, & Dekker, 2008



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What mathematical representations were students working with in the lesson?

How did Ms. Casey support students in making connections between and within different types of representations?



Facilitate meaningful mathematical discourse.

Discussions that focus on cognitively challenging mathematical tasks, namely those that promote thinking, reasoning, and problem solving, are a primary mechanism for promoting conceptual understanding of mathematics.

(Hatano & Inagaki, 1991; Michaels, O'Connor, & Resnick, 2008)



Facilitate meaningful mathematical discourse

Mathematical Discourse should:

- Build on and honor students' thinking.
- Let students share ideas, clarify understandings, and develop convincing arguments.
- Engage students in analyzing and comparing student approaches.
- Advance the math learning of the whole class.



Questions

How did Ms. Casey structure the whole class discussion to advance student learning toward the intended math learning goals?



Structuring Mathematical Discourse...

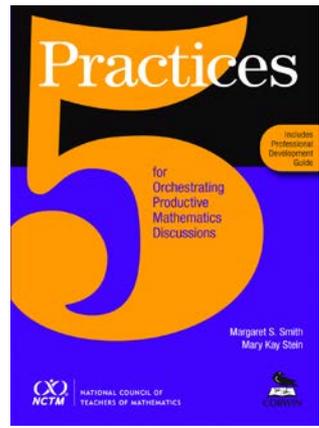
During the whole class discussion of the task, Ms. Casey was strategic in:

- Selecting specific student representations and strategies for discussion and analysis.
- Sequencing the various student approaches for analysis and comparison.
- Connecting student approaches to key math ideas and relationships.

1. Anticipating
2. Monitoring
3. Selecting
4. Sequencing
5. Connecting

5 Practices for Orchestrating Productive Mathematics Discussions

(Smith & Stein, 2011)



Pose purposeful questions.

Teachers' questions are crucial in helping students make connections and learn important mathematics and science concepts.

(Weiss & Pasley, 2004)



Pose purposeful questions

Effective Questions should:

- Reveal students' current understandings.
- Encourage students to explain, elaborate, or clarify their thinking.
- Make the targeted mathematical ideas more visible and accessible for student examination and discussion.

Boaler & Brodie, 2004; Chapin & O'Connor, 2007;
Herbel-Eisenmann & Breyfogle, 2005



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Questions

In what ways did Ms. Casey's questioning assess and advance student learning about important mathematical ideas and relationships?

Build procedural fluency from conceptual understanding.

A rush to fluency undermines students' confidence and interest in mathematics and is considered a cause of mathematics anxiety.

(Ashcraft 2002; Ramirez Gunderson, Levine, & Beilock, 2013)



Build procedural fluency from conceptual understanding

Procedural Fluency should:

- Build on a foundation of conceptual understanding.
- Over time (months, years), result in known facts and generalized methods for solving problems.
- Enable students to flexibly choose among methods to solve contextual and mathematical problems.

Baroody, 2006; Fuson & Beckmann, 2012/2013;
Fuson, Kalchman, & Bransford, 2005; Russell, 2006



Questions

In what ways did this lesson develop a foundation of conceptual understanding for building toward procedural fluency?

Support productive struggle in learning mathematics.

The struggle we have in mind comes from solving problems that are within reach and grappling with key mathematical ideas that are comprehensible but not yet well formed.

(Hiebert, Carpenter, Fennema, Fuson, Human, Murray, Olivier, & Wearne, 1996)



Support productive struggle in learning mathematics

Productive Struggle should:

- Be considered essential to learning mathematics with understanding.
- Develop students' capacity to persevere in the face of challenge.
- Help students realize that they are capable of doing well in mathematics with effort.

Black, Trzesniewski, & Dweck, 2007; Dweck, 2008;
Hiebert & Grouws, 2007; Kapur, 2010; Warshauer, 2011



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Questions

How did Ms. Casey support productive struggle among his students, individually and collectively, as they grappled with important mathematical ideas and relationships?

At which points in the lesson might Ms. Casey have consciously restrained herself from “taking over” the thinking of his students?

Elicit and use evidence of student thinking.

Teachers using assessment for learning continually look for ways in which they can generate evidence of student learning, and they use this evidence to adapt their instruction to better meet their students' learning needs.

(Leahy, Lyon, Thompson, & William, 2005, p. 23)



Elicit and use evidence of student thinking

Evidence should:

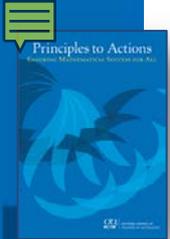
- Provide a window into students' thinking.
- Help the teacher determine the extent to which students are reaching the math learning goals.
- Be used to make instructional decisions during the lesson and to prepare for subsequent lessons.



Questions

Identify specific places during the lesson in which Ms. Casey elicited evidence of student learning and then made an instructional decision based on the evidence.

- Discuss how she used or might use that evidence to adjust his instruction to support and extend student learning.

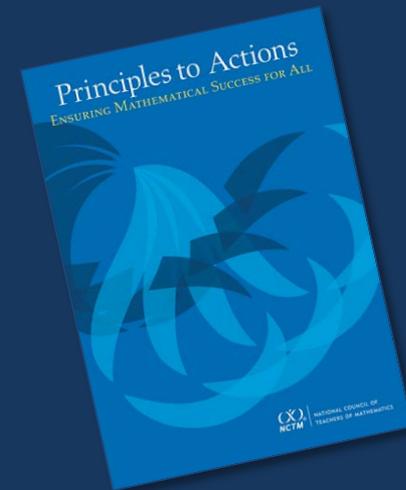


1. Work in pairs and select a string to study.
2. Identify a mathematics learning goal for the string.
3. Anticipate ways students might reason as each equation is revealed.
4. Sketch area models you would use to represent the reasoning of students.
5. If time allows, study another string.

Set A 2×8 5×8 7×8	Set B 5×4 10×4 9×4
String C 3×100 3×50 3×150 3×149	String D 200×5 200×20 200×25 198×25



Reflections and Next Steps



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Actions: Leaders and Policymakers

- Make **ongoing professional development** that supports the implementation of the eight Mathematics Teaching Practices;
- **Allocate resources** to ensure that all students are provided with an **appropriate amount of instructional time to maximize their learning potential**;
- **Structure interventions** that provide high-quality instruction and supports, such as math coaches and specialists;
- Understand the devastating impact of professional isolation and **create collaborative structures to maximize professional growth**;
- **Support risk taking and encourage new approaches** that advance student learning.

Actions: Principals, Coaches, Specialists, and other leaders

- Make the **eight Mathematics Teaching Practices** a school-wide focus that is expected for all teachers...;
- Maintain a school-wide culture with **high expectations** and a **growth mindset**;
- **Allocate time for teachers to collaborate** in professional learning communities;
- **Support improvement with multifaceted assessments** used to monitor progress and inform changes to instruction;
- Make the mathematical **success of every student a nonnegotiable priority.**

Actions: Teachers

- Plan and implement effective instruction as described by the Mathematics Teaching Practices;
- Develop socially, emotionally, and academically safe environments for mathematics teaching and;
- Evaluate curricular materials and resources to determine the extent to which these materials align with the standards, ensure coherent development of topics within and across grades...;
- Work collaboratively with colleagues to plan instruction, solve common challenges, and provide mutual support as they take collective responsibility for student learning.



<http://www.nctm.org/PtAToolkit/>

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