VIRGINIA DEPARTMENT OF EDUCATION

VDOE Spring 2017 Mathematics Institute Algebra I

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Welcome and Introductions

Teachers? Math Specialists? Administrators? Central Office?

Elementary? Middle? High?

Agenda

I. Revisions to Standards and Purpose

II. Emphasis on Specific Content

   A. Transformations

   B. Graphing Linear Equations

   C. Multiple Representations of Functions

III. Support for Implementation

I. Revisions to Standards and Purpose

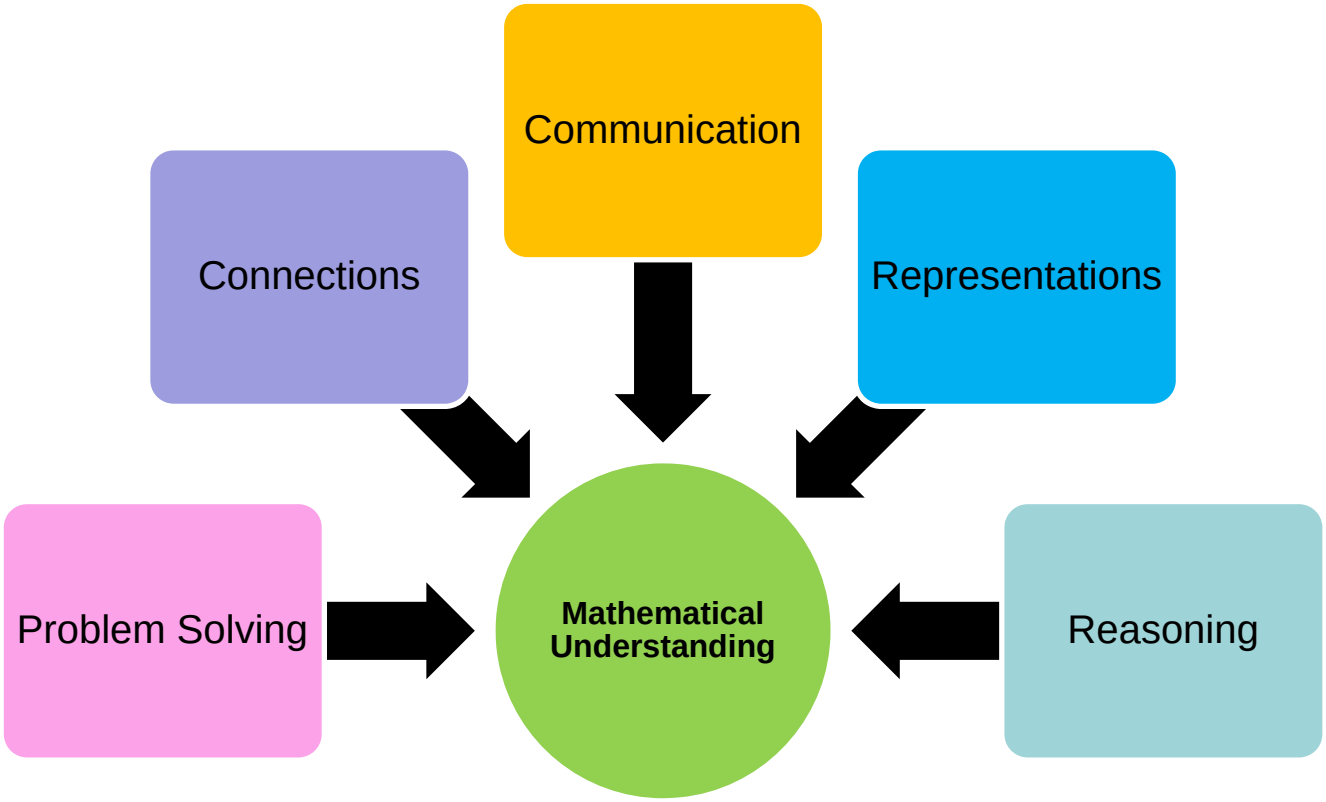
Essential Question

- What are the new 2016 Standards of Learning and how might the VDOE documents support understanding of these standards?

Mathematics Process Goals for Students

"The content of the mathematics standards is intended to support the five process goals for students"

- 2009 and 2016 *Mathematics Standards of Learning*



NCTM *Principles to Actions*
Ensuring Mathematical Success for All

High Leverage Mathematics Teaching Practices
1. Establish mathematics goals to focus learning.
2. Implement tasks that promote reasoning and problem solving.
3. Use and connect mathematical representations.
4. Facilitate meaningful mathematical discourse.
5. Pose purposeful questions.
6. Build procedural fluency from conceptual understanding.
7. Support productive struggle in learning mathematics.
8. Elicit and use evidence of student thinking.

Adapted from Leinwand, S. et al. (2014) *Principles to Actions – Ensuring Mathematical Success for All,* National Council of Teachers of Mathematics.

Changes to the Curriculum Framework

- Indicators of SOL sub-bullet added to each bullet within the Essential Knowledge and Skills

A.1 **The student will**
a) represent verbal quantitative situations algebraically; and
b) evaluate algebraic expressions for given replacement values of the variables.

Understanding the Standard	Essential Knowledge and Skills
• Mathematical modeling involves creating algebraic representations of quantitative practical situations. • The numerical value of an expression depends upon the values of the replacement set for the variables. • There are a variety of ways to compute the value of a numerical expression and evaluate an algebraic expression using order of operations. • The operations and the magnitude of the numbers in an expression affect the choice of an appropriate computational technique (e.g., mental mathematics, calculator, paper and pencil).	**The student will use problem solving, mathematical communication, mathematical reasoning, connections, and representations to** • Translate between verbal quantitative situations and algebraic expressions and equations. (a) • Represent practical situations with algebraic expressions in a variety of representations (e.g., concrete, pictorial, symbolic, verbal). (a) • Evaluate algebraic expressions, using the order of operations, which include absolute value, square roots, and cube roots for given replacement values to include rational numbers, without rationalizing the denominator. (b)

Overview of Crosswalk Documents

- By grade level/course
- Includes:
  - Additions
  - Deletions
  - Parameter changes/ clarifications (2016 SOL)
  - Moves within the grade level (2009 SOL to 2016 SOL)

Additions (2016)	**Deletions (2009)**
Parameter Changes (2016)	**Moves (2009 to 2016)**

Geometry – Crosswalk (Summary of Revisions): 2016 *Mathematics Standards of Learning and Curriculum Framework*

Additions (2016 SOL)	Deletions from Geometry (2009 SOL)
• G.1c EKS – Determine that an argument is false using a counterexample • G.2b EKS – Added same-side exterior angles to the required relationships between pairs of angles • G.3a EKS – Use the midpoint formula to determine the endpoint of a segment in addition to the midpoint • G.10b EKS – Determine angle measures of a regular polygon in a tessellation • G.13 EKS – Determine surface area and volume of hemispheres • G.13 EKS – Solve problems involving lateral area of cylinders, prisms, and regular pyramids	• G.1c – Use Venn diagrams to represent set relationships; interpret Venn diagrams [Moved to DM.12] • G.4 EKS – Construct the inscribed and circumscribed circles of a triangle; and a tangent line from a point outside a given circle to the circle
Parameter Changes/Clarifications (2016 SOL)	**Moves within Geometry (2009 SOL to 2016 SOL)**
• G.6, G.7, and G.9 – proofs are referred to as "direct proofs" and the following statement was added to the US: "Deductive or inductive reasoning is used in mathematical proofs. In this course, deductive reasoning and logic are used in direct proofs. Direct proofs are presented in different formats (typically two-column or paragraph) and employ definitions, postulates, theorems, and algebraic justifications including coordinate methods" • G.2a – Verifying parallelism (formerly G.2b) included under "prove" • G.3d EKS –Clarified that transformations may be a combination of transformations, and includes a list of possible transformations; reflections limited to reflections over any horizontal line, vertical line, the line $y = x$, or the line $y = -x$; rotations limited to 90°, 180°, 270°, or 360° on a coordinate grid where the center of rotation at the origin; dilations limited to those from a fixed point on the coordinate grid • G.8b EKS – Clarified that solving problems for missing lengths in right triangles may include situations where rationalizing denominators may be necessary • G.8c EKS – Clarified that solving problems involving right triangles may include determining missing side lengths or angle measurements • G.10a,b,c – Clarified that solving problems involves angles of convex polygons • G.12 EKS – Clarified expectations for solving problems involving equations of circles • G.13 US – Clarified types of cylinders, cones, prisms, and pyramids used in solving practical problems involving volume and surface area	• G.1d – [Moved to standard stem] • G.1 – Determine the validity of a logical argument [Moved from standard stem to G.1c] • G.2c – [Moved to G.2b] • G.4h – [Moved from EKS] • G.11a – [Moved to standard stem] • G.11b – [Moved to standard stem] • G.11c – Area of a sector [Moved to G.11d]

EKS = Essential Knowledge and Skills, referring to the column on the right side of the Curriculum Framework
US = Understanding the Standard, referring to the column on the left side of the Curriculum Framework

Grade Level Crosswalk (Summary of Revisions)

Algebra 1 Course Introductions – Part 1

In order to assist students in developing meaning and connecting algebraic concepts to geometry and statistics, consideration should be given to the sequential development of concepts and skills by using **concrete materials** to assist students in making the transition from the numeric to the symbolic.

Connections between Algebra I and other subject areas through **practical applications** may assist in helping students attach meaning to the abstract concepts of algebra.

Algebra 1 Course Introductions – Part 2

Tables and graphs will be used to interpret algebraic expressions, equations, and inequalities and to analyze behaviors of functions. **(Multiple Representations)**

These standards include a **transformational approach** to graphing functions and writing equations when given the graph of the equation. Transformational graphing builds a strong connection between algebraic and graphic representations of functions.

Graphing utilities (calculators, computers, and other technology tools) will be used to assist in teaching and learning. Graphing utilities facilitate visualizing, analyzing, and understanding algebraic and statistical behaviors and provide a powerful tool for solving and verifying solutions.

Global Changes

Turn and Talk

- Discuss the key topics in the Algebra 1 Course Introduction.
- Why do you think the changes were made?

Properties of Real Numbers and Equality/Inequality

Algebra I:

- 2009. A.4: Justify steps used in simplifying expressions and solving equations using field properties
- 2016. A.4 EKS: Apply the properties of real numbers...to simplify expressions and solve equations

Algebra II:

- 2009. A.II.3: Identify field properties that are valid for complex numbers.
- 2016: N/A

Linear Equations and Inequalities

6.13: One-step linear equations
6.14: One-step linear inequality (addition/subtraction only)

7.12: Two-step linear equations
7.13: One- and two-step linear inequalities

8.17: Multi-step linear equations
8.18: Multi-step linear inequalities

A.4: Multi-step linear equations
A.5: Multi-step linear inequalities

Linear Equations and Inequalities

From Math 8 Essential Knowledge and Skills:

- Variable on one or both sides of the equation
- Coefficients and numeric terms will be rational
- May contain expressions that need to be expanded using the distributive property
- May require collecting like terms to solve
- Up to four steps

Essential Question Recap

- What are the new 2016 Standards of Learning and how might the VDOE documents support understanding of these standards?

2a. Transformations

Essential Questions

- What instructional strategies promote students' understanding of transformations using technology?

Transformations

- <u>Algebra I Number talk</u>
  Tell me everything you can about the graph of $y = 2x + 3$.

Transformations

- Desmos Demonstration -- Function Transformation: Practice with Symbols

Teacher page: https://teacher.desmos.com
Student page: https://student.desmos.com

Class code: 6W6VS

Essential Questions Recap

- What instructional strategies promote students' understanding of transformations using technology?

2b. Graphing Linear Equations

Essential Question

- How do the 2016 K-8 progressions impact the way we teach graphing linear equations in Algebra I?

Sort Activity

With a partner:

- Each slip has one of the Essential Knowledge and Skills for the 2016 PFA progression.
- Place each skill with the grade level (Grade 5 – Geometry) where you think it belongs.
- Share your answers with another group.
- Are there any discrepancies?

Sort Activity

PFA Proportional Reasoning Progression Sort					
Grade 5	Grade 6	Grade 7	Grade 8	Algebra 1	Geometry

				Algebra 1
• Identify, c patterns lines, tabl	• Make a ta represent between practical • Determin relationsh given a gr are limite	• Determin in a propo quantities verbal des represent write an e represent limited to • Graph a li relationsh the equat $y = mx$, v rate of ch positive va • Graph a li relationsh the equat The y-inte values and	• Recognize that is po • Given a li $y = mx$ - y-interce • Write the the form slope, m, practical and y-inte	• Determine the slope of the line, given the equation of a linear function. • Write the equation of a line when given the graph of a line. • Write the equation of a vertical line as $x = a$. • Write the equation of a line parallel or perpendicular to a given line through a given point.

Algebra (Proportional Reasoning) Progression

SOL 6.12	SOL 7.10	SOL 8.16
Essential Knowledge and Skills	**Essential Knowledge and Skills**	**Essential Knowledge and Skills**
The student will use problem solving, mathematical communication, mathematical reasoning, connections, and representations to	**The student will use problem solving, mathematical communication, mathematical reasoning, connections, and representations to**	**The student will use problem solving, mathematical communication, mathematical reasoning, connections, and representations to**
• Mal rep bet (a) • Make a table of equivalent ratios to represent a **proportional relationship** between two quantities, when given a pr • Id re or re ra • Determine a missing value in a ratio table that represents a **proportional relationship** between two quantities using a unit rate. Unit rates are limited to positive values. (b) • Determine whether a **proportional relationship** exists between two quantities, when given a table of values or a verbal description, including those represented in a practical situation. Unit rates are limited to positive values. (c) • Determine whether a **proportional relationship** exists between two quantities given a graph of ordered pairs. Unit rates are limited to positive values. (c) • Make connections between and among multiple representations of the same **proportional relationship** using verbal descriptions, ratio tables, and graphs. Unit rates are limited to positive values. (d)	practical situation, and write an equation in the form $y = mx$ to represent the relationship. Slope will be limited to positive values. (a) • Graph a line representing a **proportional relationship**, Slope will be limited to positive values. (b) • Determine the y-intercept, b, in an **additive relationship** between two quantities given a table of values or a verbal description, including those represented in a practical situation, and write an equation in the form $y = x + b$, $b \neq 0$, represent the relationship. (c) • Graph a line representing an **additive relationship** ($y = x + b$ $b \neq 0$) between two quantities, given an ordered pair on the line and the y-intercept (b). The y-intercept (b) is limited to integer values and slope is limited to 1. (d) • Graph a line representing an **additive relationship** between two quantities, given the equation in the form $y = x + b$, $b \neq 0$. The y-intercept (b) is limited to integer values and slope is limited to 1. (d) • Make connections between and among representations of a **proportional or additive relationship** between two quantities using verbal descriptions, tables, equations, and graphs. (e)	ize and describe a line with a slope positive, negative, or zero (0). (a) table of values for a **linear function**, identify the slope and y-intercept. The table will include the coordinate of the y-intercept. (b) • Given a **linear function** in the form $y = mx +$ slope and y-intercept. (b) h of **a linear function**, pe and y-intercept. The intercept will be limited to coordinates of the ordered pairs shown in the graph will be limited to integers. (b) • Identify the dependent and independent variable, given a practical situation • Write the equation of a **linear function** in the form $y = mx + b$ given values for the slope, m, and the y-intercept or given a practical situation in which the slope, m, and y-intercept are described verbally. (e) • Make connections between and among representations of a **linear function** using verbal descriptions, tables, equations, and graphs. (e).

PROPORTIONAL RELATIONSHIPS

NON-PROPORTIONAL/ADDITIVE RELATIONSHIPS

LINEAR FUNCTIONS

Algebra (Proportional Reasoning) Progression

SOL 6.12	SOL 7.10	SOL 8.16
Essential Knowledge and Skills	**Essential Knowledge and Skills**	**Essential Knowledge and Skills**
The student will use problem solving, mathematical communication, mathematical connections, and representatio	**The student will use problem solving, mathematical**	**The student will use problem solving, mmunication, mathematical ctions, and representations to**
• Make a table of equivalen represent a **proportional relationship** between two quantities, when given a ratio. (a)	**relationship** between two quantities given a table of values or a verbal description, including those represented in a practical situation, and write an equation in the form $y = mx$ to represent the relationship. Slope will be limited to positive values. (a)	and describe a line with a slope that is positive, negative, or zero (0). (a)
• between two quantities, when given a practical situation. (a)	• Graph a line representing a **proportional relationship**, between two quantities given an ordered pair on the line	• Given a table of values for a **linear function**, identify the slope and y-intercept. The table will include the coordinate of the y-intercept. (b)
• Identify the unit rate of a **proportiona relationship** represented by a table o or a verbal description, including thos represented in a practical situation. Unit rates are limited to positive values. (b)	between two quantities given the equation of the line in the form $y = mx$, where m represents the slope as rate of change. Slope will be limited to positive va	• Given a **linear function** in the form $y = mx +$ tify the slope and y-intercept. (b)
• Determine a missing value in a ratio table that represents a **proportional relationship** between two quantities using a unit rate. Unit rates are limited to positive values. (b)	• Determine the y-intercept, b, in a between two quantities given a ta description, including those represented in a practical situation, and write an equation in the form $y = x + b$, $b \neq 0$, to represent the relationship. (c)	the graph of a **linear function**, y the slope and y-intercept. The value of the y-intercept will be limited to integers. The coordinates of the ordered variable, given a practical situation modeled by a **linear function**. (c)
• Determine whether a **proportional relationship** exists between two quantities, when given a table of values or a verbal description, including those represented in a practical situation. Unit rates are limited to positive values. (c)	• Graph a line representing an **additive relationship** ($y = x + b$, $b \neq$ r on the line ited to inte	• Given the equation of a **linear function** in the form $y = mx + b$, graph the function. The value of the y-intercept will be limited to integers. (d)
• Determine whether a **proportional relationship** exists between two quantities given a graph of ordered pairs. Unit rates are limited to positive values. (c)	• Gra etween two + b, $b \neq 0$. The y-intercept (b) is limited to integer values and slope is limited to 1. (d)	• practical situation in which the slope, m, and y-intercept are described verbally. (e)
• Make connections between and among multiple representations of the same **proportional relationship** using verbal descriptions, ratio tables, and graphs. Unit rates are limited to positive values. (d)	• Make connections between and among representations of a **proportional or additive relationship** between two quantities using verbal descriptions, tables, equations, and graphs. (e)	• Make connections between and among representations of a **linear function** using verbal descriptions, tables, equations, and graphs. (e).

RATIO TABLES

UNIT RATES

SLOPE as RATE of CHANGE

SLOPE and y-INTERCEPT

$$y = mx$$

$$y = x + b$$

$$y = mx + b$$

Graphing Linear Equations

- Compare 2009 and 2016 Standards. What do you notice?

2009 SOL
A.6 The student will graph linear equations and linear inequalities in two variables, including a) determining the slope of a line when given an equation of the line, the graph of the line, or two points on the line. Slope will be described as rate of change and will be positive, negative, zero, or undefined; and b) writing the equation of a line when given the graph of the line, two points on the line, or the slope and a point on the line.

2016 SOL
A.6 The student will a) determine the slope of a line when given an equation of the line, the graph of the line, or two points on the line; b) write the equation of a line when given the graph of the line, two points on the line, or the slope and a point on the line; and c) graph linear equations in two variables.

Graphing Linear Equations

- Look at the progression document.
- Discussion:
  Algebra I expectations are the same but students are coming with very different experiences. How does our instruction need to be adapted?

Essential Question Recap

- How do the K-8 progressions impact the way we teach graphing linear equations in Algebra I?

2c. Multiple Representations of Functions

Essential Question

- What instructional strategies promote students' flexibility with multiple representations of functions?

Vertical Progressions - Sorting

Sort the problems by the grade level (K-12) in which you think they are covered.

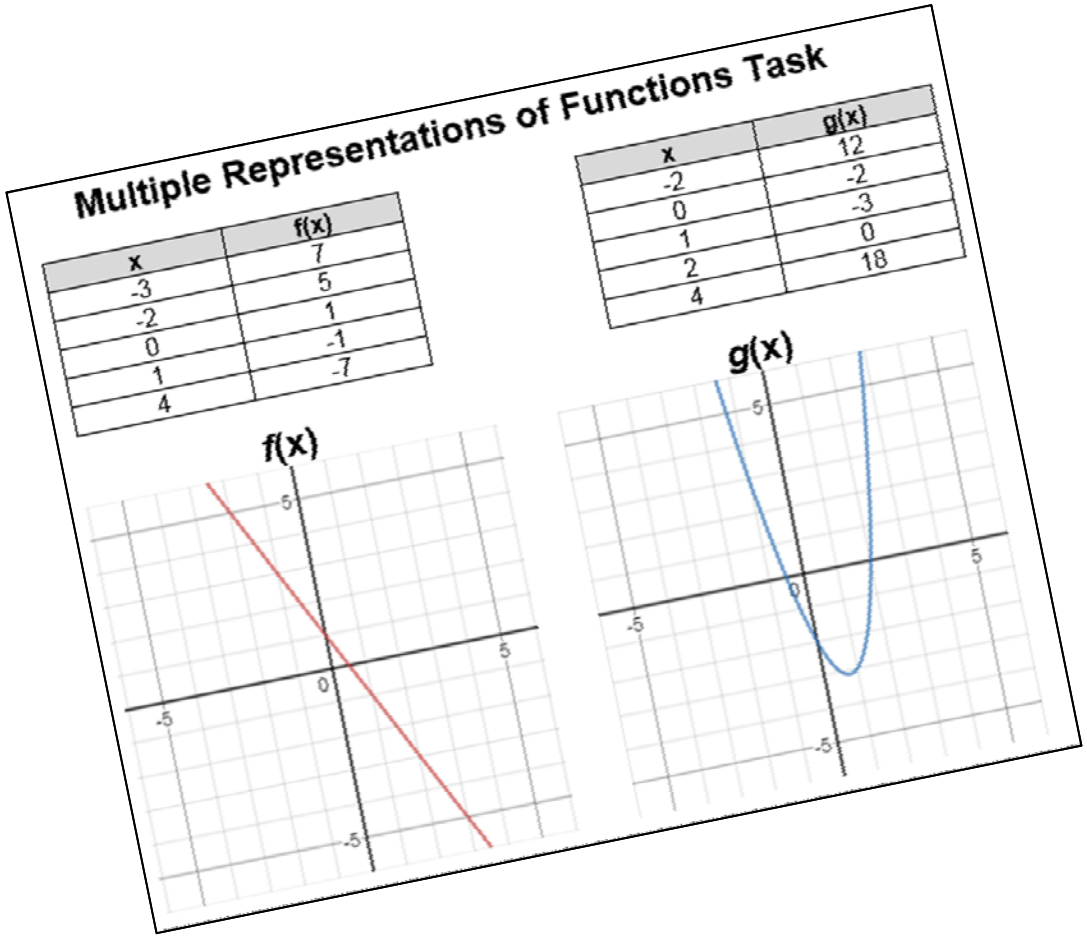
Multiple Representations of Functions

A.7 EKS: Investigate and analyze characteristics and multiple representations of functions with a graphing utility.

AII.7 EKS: Investigate and analyze characteristics and multiple representations of functions with a graphing utility.

Multiple Representations of Functions

- Complete the task with a partner.



Multiple Representations of Functions Task

x	f(x)
-3	7
-2	5
0	1
1	-1
4	-7

x	g(x)
-2	12
0	-2
1	-3
2	0
4	18

Multiple Representations of Functions

- How does the use of multiple representations in this task inform you of student understanding?
- How does it demonstrate the need for multiple representations?
- How can you modify these problems to change the cognitive demand?

Essential Question Recap

- What instructional strategies promote students' flexibility with multiple representations of functions?

Essential Questions

- What resources are available to help implement the 2016 Standards?
- Where do we go from here?

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