Mathematical Modeling

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1966-2016 Celebrating 50 Years

MSIC

MathScience Innovation Center

Imagine. Create. Lead.

Snack Bag Math

At our school's upcoming carnival fundraiser, your class has be asked to run a concession stand and sell snack bags. We know from experience that about 100 people are likely to attend.

The school will buy the ingredients from a local vendor who sells them in bulk, but we have a

maximum budget of $65.00. Below are three available ingredients, along with their cost:

- Pretzels: $2.72/pound ($0.17/ounce)
- Raisins: $3.52/pound ($0.22/ounce)
- Chocolate chips: $4.64/pound ($0.29/ounce)

The snack bags, which we already have in stock, will hold no more than about four ounces of ingredients.

The Task

You will write a proposal to the fundraising committee that includes the following information:

- A snack bag “recipe” (You may decide to make more than one type of combination.) Explain why you chose the combination (s) of ingredients.

- What the committee will need to buy in advance from the local bulk food store.

- Proposed sale price of the snack bag(s). Please explain why you have decided on a particular sale price.

Definition

- A **mathematical model** is a representation in mathematical terms of the behavior of real devices or phenomena.

Definition

Mathematical modeling is developing the mathematical representations

and then validating the representation

and using the representation to explain the behavior of the object

and make predictions about the object.

Who needs them?

Mathematical models are used in

- the natural sciences (i.e. physics, biology, earth science, meteorology),
- engineering disciplines (i.e. computer science, psychology, sociology, political science),
- the business and medical fields.

Why?

- A model may help to explain a system, to study the effects of different components of the system, and to make predictions about the system's behavior.

How do we prepare students to do this type of work?

The Modeling Cycle

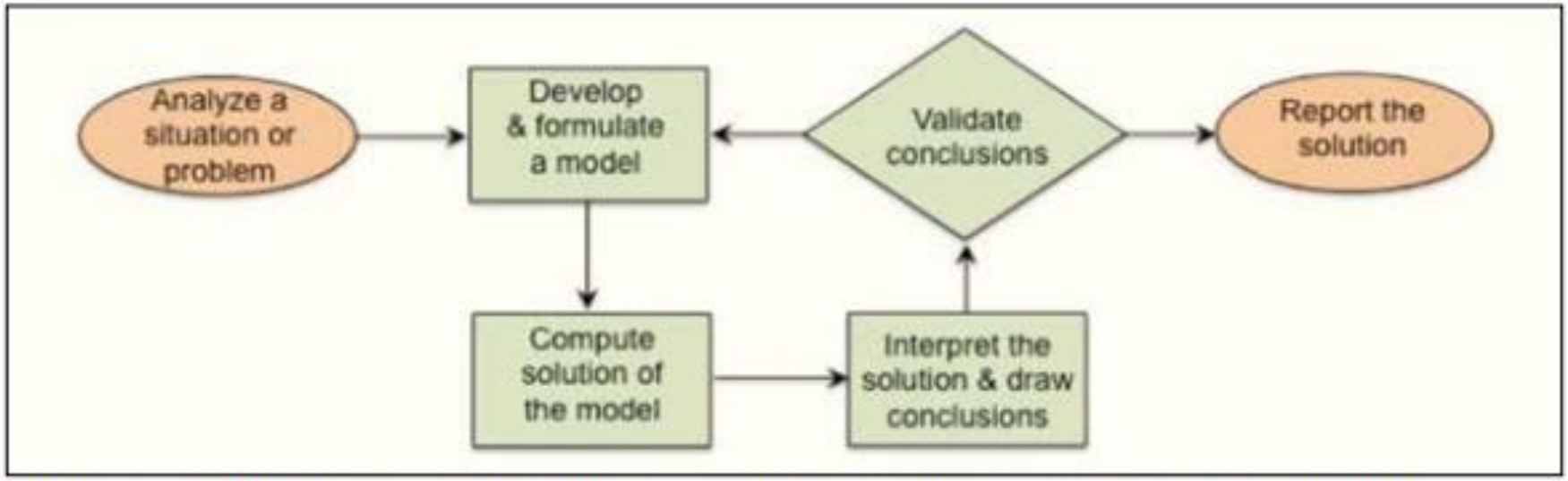


Fig. 1 Element titles within the modeling cycle (CCSSI 2010, p. 72) have been expanded to be more descriptive.

Anhalt, Cynthia O., and Ricardo Cortez. "Modeling: A Structured Process." *Mathematics Teacher* 108.6 (2015): 447-52.

Analyze the Problem or Situation

- Identify a problem taken from an external context (often from an everyday life context) that must be solved or a situation that must be understood and explained.
- Do background research if necessary.
- Make sense of the situation or problem and understand the question

Develop and formulate a model

- Determine all given information.

- Determine what assumptions are necessary.

- Translate the information given in the problem together with the assumptions into a mathematical problem that can be solved.

- Use mathematics appropriate for the information given and assumed as well as the students' expertise.

Compute a solution of the model

- Solve the mathematical problem stated in the model.
- Analyze and perform operations in the model.
- Check for correctness.

Interpret the solution and draw conclusions

- Interpret the mathematical solution in terms of the original situation

- Draw conclusions that the solution implies about the original situation

Validate conclusions

- Reflect on whether the mathematical answer makes sense in terms of the original situation (e.g., is the answer within a valid range of values?).

- If the conclusions are satisfactory with regard to the accuracy needed, report the solution. If the conclusions are not satisfactory or need to be improved, go back to stage 2 (“Develop and formulate a model”).

Develop and formulate a new or modified model

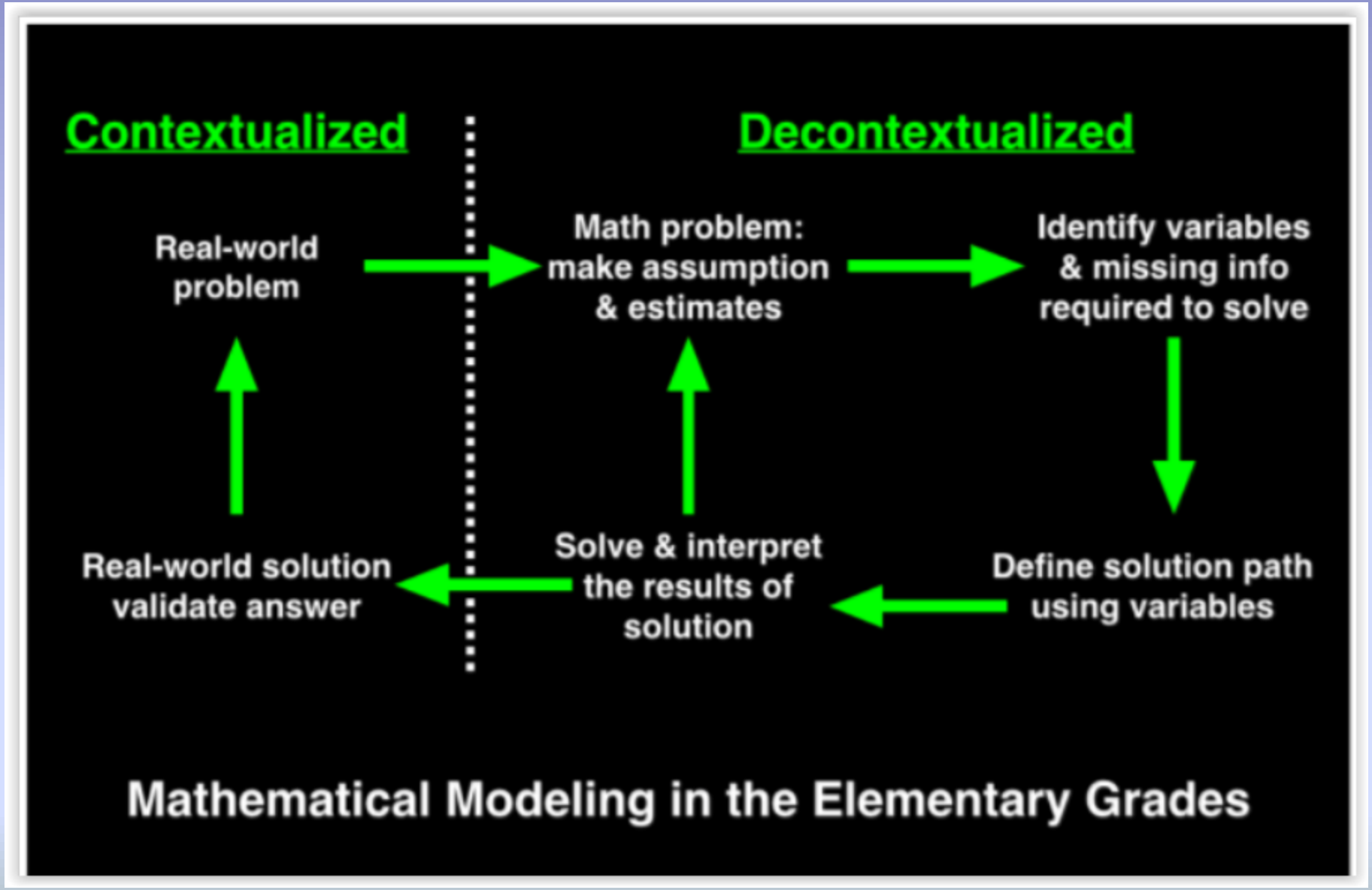
- Revise the assumptions made according to what was learned in the first solution and translate them into a new or modified mathematical problem that can be solved.

- The type of mathematics in the current model may be different from the previous one.

- Go through these stages again: Compute, Interpret, and Validate

Report the solution

- Share your conclusions and the reasoning behind them

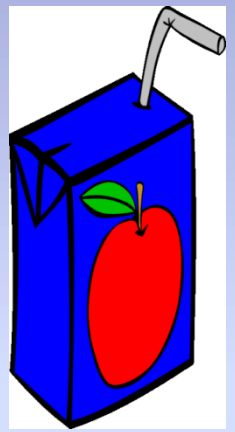


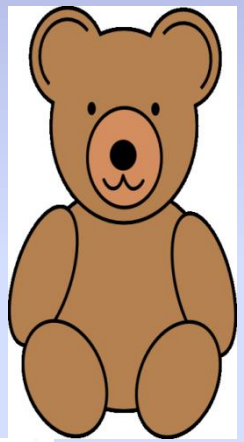
https://gfletchy.com/2015/08/28/get-your-model-on-modeling-in-the-elementary-grades/

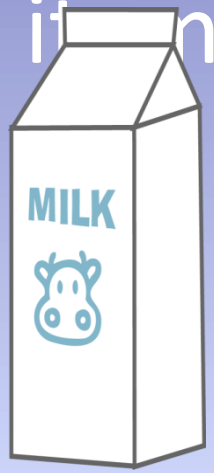
Baxter Brown's Messy Room

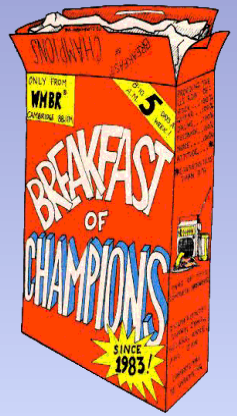
Lyn English

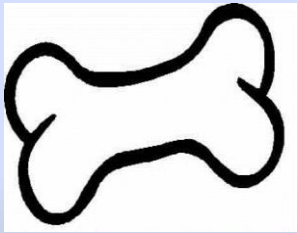
What are some of the important attributes of these items?









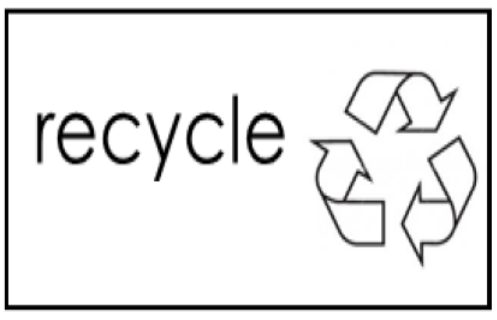


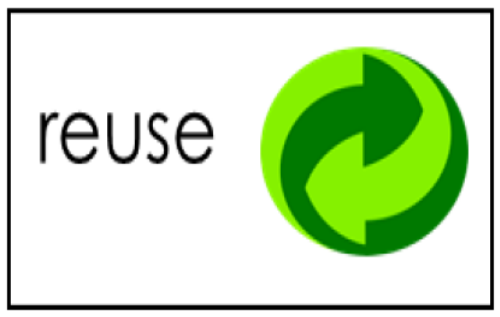
Question

Baxter Brown needs help with cleaning up his messy room. Help him decide which objects can be recycled, reused, or thrown away.

Organize the items in any way you'd like into these three categories. Explain your reasoning.

throw
away


recycle


reuse

Children's Group 1 Sort

Recycle	Reuse	throw away
milk carton	Toy	apple core
rubbish bag	Newspaper	drink can
cereal packet	Bone	
	Shoe	
	dog biscuit	

Children's Group 2 Sort

Recycle	Reuse	throw away
Bone	Toy	apple core
dog biscuit	rubbish bag	cereal packet
drink can	Shoe	
milk carton		
Newspaper		

Children's Group 3 Sort

Recycle	Reuse	throw away
drink can	cereal packet	Bone
apple core	Shoe	Toy
milk carton	newspaper	Dog biscuit
rubbish bag		
Newspaper		

Children's Group 4 Sort

Recycle	Reuse	throw away
cereal packet	bone	rubbish bag
dog biscuit	newspaper	drink can
milk carton	shoe	Dog biscuit
Toy		
apple core		

The Annual Paper Airplane Contest

This year the Stoney Creek Flight School will hold their annual paper airplane flying contest on September 15. Students in year three will be working in groups and will design one plane.

All planes will be designed to fly for as long as possible in the air (time) and over a long distance from a target. The plane will need to travel in a straight-line path.

Three awards will be given at this contest. One will be given to the group whose plane stays in the air the longest; another to the group whose plane travels the longest straight-line path, and the final award is an overall award given to the group who wins the contest.

Investigation

In the past years, the judges have had problems with deciding how to select a winner and how to judge the contest. Using the given data from the previous years, find a way to help the judges decide on the overall winner of the contest.

Write a letter to the judges of the contest explaining to them how to determine who wins each of the categories (time in the air and distance travelled in a straight-line path) and how to decide the winner of the overall award for the contest.

The Better Box Problem

The dimensions of a typical cereal box measure approximately 2 in wide x 6 in long x 12 in high.

The design of a cereal box has not changed much over many years.

The Better Box Problem

Jenna is always looking for ways to recycle and reuse the paperboard used to construct the box because she cares about the environment.

She wonders if there is a better way to package cereal using less paper but maintaining the same volume. Can you help Jenna?

The Better Box Problem

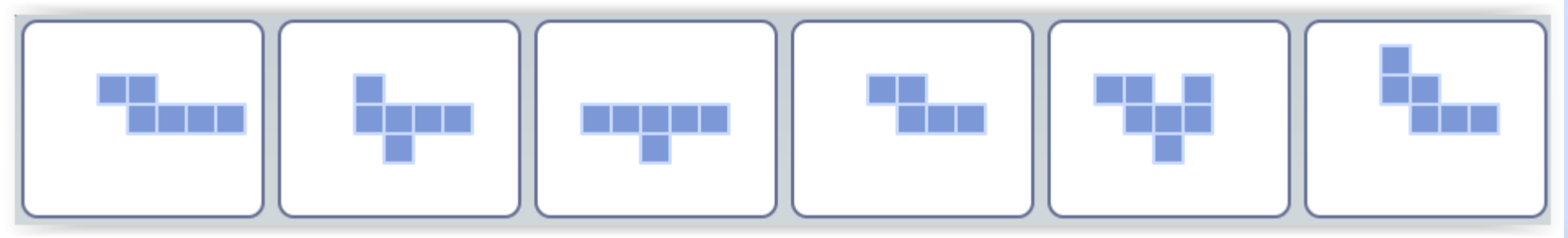
- Is there a better cereal box that would have the same volume as the original cereal box but use less paper?
- How much paper is saved?
- What other factors might a cereal company take into consideration when considering a change in packaging dimensions. How does each box address these?

The Better Box Problem

What experiences do students need to have prior to being assigned The Better Box problem?

Better Box Problem: Potential Prior Experiences

- Video on packaging design
- Find volume of various packages
- Making a net of various packaging and finding surface area
- Illuminations activity: http://illuminations.nctm.org/ActivityDetail.aspx?ID=84



- Design a package for something else

How do you grade a modeling problem?

Table 2 Mathematical Modeling Task Categories for Evaluating		
Categories for Scoring	**Score**	**Observations and Comments**
Reasonable and relevant assumptions		
Formulation of a model based on assumptions		
Solution of the model		
Interpretation of solution		
Validation of conclusions		
Modeling cycle—revised assumptions and model		
Reflection on thought process as the model was created and the modeling cycle considered		
Reflection on other ways to model this problem besides the approach taken		
	Total	

Table 3 A General Rubric for Assessment of Modeling Problems

	5	4	3	2	1	0
Explanations	Demonstrates full understanding and provides full explanations: justifications and explanations demonstrate understanding of concepts	Demonstrates basic understanding and provides minimal explanations	Demonstrates some understanding but has some gaps, brief explanations	Demonstrates little understanding, gaps in thinking, little to no explanations	Demonstrates some partial understanding with no explanation	Shows no evidence of understanding
Connections	Shows ideas that are well-connected: uses more than one concept and shows understanding of their connection	Shows ideas that are well connected	Shows ideas that are partially connected, missing points, or unclear	Shows ideas that are not well connected	Shows ideas that are not well connected	
Work	Provides complete work: includes assumptions and solutions that follow them	Provides complete work	Provides incomplete work but includes essential information	Provides incomplete work, missing some essential information	Provides incomplete work, missing essential information	Shows no evidence of work
Reasoning	Shows evidence of thoughtfulness and reasoning	Shows evidence of reasoning	Shows evidence of partial reasoning	Shows little evidence any reasoning	Shows evidence of faulty reasoning	
Concepts	Displays correct conceptual mathematical ideas: uses appropriate and multiple representations, uses appropriate concepts for the given problem	Displays correct conceptual mathematical ideas	Displays partial correct conceptual mathematical ideas	Displays incorrect conceptual mathematical ideas	Displays incorrect conceptual mathematical ideas	
Calculations	Presents correct calculations (possibly one minor error): numerical calculations are correct, appropriate units are used	Presents minor errors in calculations	Presents consequential errors in calculations	Presents significant errors in calculations	Presents significant errors in calculations	

Resources

- Your students!

  -John's Soccer Cleat Dilemma

  -Designing a Recess Area

- NCTM Publications
- Graham Fletcher: gfletchy.com/3-act-lessons/
- GAIMME Report (Guidelines for Assessment and Instruction in Mathematical Modeling Education)