

Linear Function: A Stack of Cups Algebra Lesson Plan

Overview: In this lesson, students describe a relationship between the number of cups and the height of the stack of cups.

Mathematics: To solve this task successfully, students need to understand that a linear relationship is defined by a rate of change that is constant. The essential characteristic of linear functions is that they have a *constant rate of change*. The *change* in quantity y is a *constant multiple* of the *change* in quantity x .

$$\frac{\text{change in } y}{\text{change in } x} = k$$

A linear function can be represented using a table, a formula, a graph, a diagram, or a verbal description. What is essential is identifying the constant rate of change in each representation. These representations vary in usefulness and so it is also important to be able to move fluently from any one of these representations to any other.

Goals:

- Students will solve the problem using a variety of strategies.
- Students will develop an understanding of linear function.
- Students will demonstrate an understanding of constant rate of change.
- Students will graph the relationship of the number of cups and the height of the stack of cups.
- Students will explain the meaning of slope in the relationship of the stack of cups.
- Students will justify their solutions to the problem.

Algebra Standards:

- 4.0 Students simplify expressions prior to solving linear equations and inequalities in one variable, such as $3(2x-5)+4(x-2)=12$
- 5.0 Students solve multi-step problems, including word problems, involving linear equations and linear inequalities in one variable and provide justification for each step.

Materials: Stack of cups (attached); chart paper; graph paper; markers

Access Strategies: Throughout the document you will see icons calling out use of the access strategies for English Learners, Standard English Learners, and Students With Disabilities.

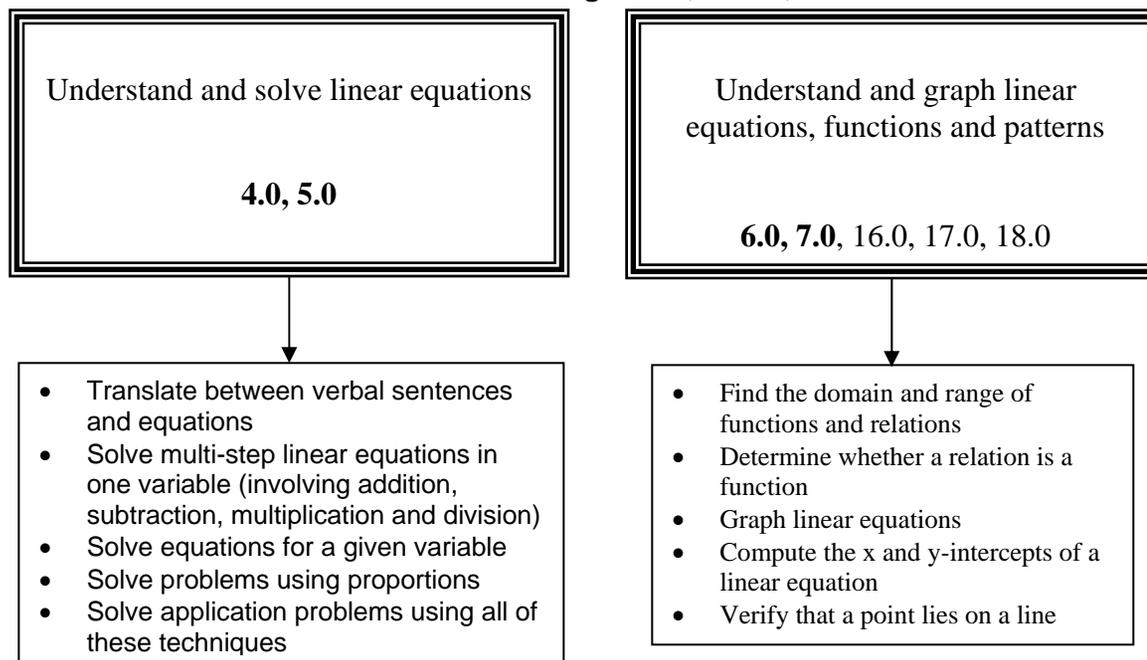
Access Strategy	Icon	Description
Cooperative and Communal Learning Environments		<p><i>Supportive learning environments that motivate students to engage more with learning and that promote language acquisition through meaningful interactions and positive learning experiences to achieve an instructional goal. Working collaboratively in small groups, students learn faster and more efficiently, have greater retention of concepts, and feel positive about their learning.</i></p>
Instructional Conversations		<p><i>Discussion-based lessons carried out with the assistance of more competent others who help students arrive at a deeper understanding of academic content. ICs provide opportunities for students to use language in interactions that promote analysis, reflection, and critical thinking. These classroom interactions create opportunities for students' conceptual and linguistic development by making connections between academic content, students' prior knowledge, and cultural experiences</i></p>
Academic Language Development		<p><i>The teaching of specialized language, vocabulary, grammar, structures, patterns, and features that occur with high frequency in academic texts and discourse. ALD builds on the conceptual knowledge and vocabulary students bring from their home and community environments. Academic language proficiency is a prerequisite skill that aids comprehension and prepares students to effectively communicate in different academic areas.</i></p>
Advanced Graphic Organizers		<p><i>Visual tools and representations of information that show the structure of concepts and the relationships between ideas to support critical thinking processes. Their effective use promotes active learning that helps students construct knowledge, organize thinking, visualize abstract concepts, and gain a clearer understanding of instructional material.</i></p>

Academic Language Goals of the Lesson:

- Develop academic vocabulary to be used in the descriptions
- Describe algebraic patterns orally or in writing
- Explain the process used in solving the task, orally or in writing

<p>Assumption of Prior Knowledge:</p> <ul style="list-style-type: none"> • Plotting points on the coordinate plane • Measuring lengths • Using variables to represent real-world quantities 	<p>Academic Language:</p> <ul style="list-style-type: none"> • Slope • Constant rate of change • Linear • Relationship • Y-intercept 	<p>Materials:</p> <ul style="list-style-type: none"> • Task • Styrofoam cups (at least 5 per group) • Chart Paper • Marker pens 	<p>Follow-Up Lessons:</p> <ul style="list-style-type: none"> • Procedures for Multiplying Binomials, FOIL and box methods, and how they relate to using algebra tiles
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Connections to the LAUSD Algebra 1, Unit 1, Instructional Guide



CLE	Students work in groups as they investigate the stack of cups	ALD	Use reading strategies to assist students understand the task
SETUP	<p style="text-align: center;">Action</p> <p>Prior to the lesson:</p> <ul style="list-style-type: none"> • Arrange the desks so that students are in groups of 4. • Determine student groups prior to the lesson so that students who complement each other's skills and knowledge core are working together. • Place materials for the task at each grouping. • Solve the task yourself. <p>HOW DO I SET-UP THE LESSON? Ask students to follow along as you read the situation presented in the Stack of Cups task. Then have several students explain to the class what they are trying to find when solving the problem. Stress to students that they will be expected to explain how and why they solved the problem a particular way and to refer to the context of the problem.</p> <p><i>To assist English learners/Standard English learners in getting into the task and understanding what they are to do, you may want to use one of the following strategies: Think, Pair, Share, in cooperative groups, and Verbalizing (asking questions to check for understanding).</i></p>	<p style="text-align: center;">Comments</p> <p>Students will be more successful in this task if they understand what is expected in terms of group work and the final product.</p> <p>It is critical that you solve the problem in as many ways as possible so that you become familiar with strategies students may use. This will allow you to better understand students' thinking. As you read through this lesson plan, different questions about the problem will be given.</p> <p>HOW DO I SET-UP THE LESSON? As students describe the task, listen for their understanding of the goals of the task. It is important that they indicate the goal is to determine and demonstrate the relationship between the size of the cups and the height of the stack of the cups. Be careful not to tell students how to solve the task or to set up a procedure for solving the task because your goal is for students to do the problem-solving.</p> <p><i>Using Think, Pair, Share in cooperative groups will support and assist students (EL/SEL) to engage with the task and interact collaboratively with their peers to clarify, explain, question, etc. what they are being asked to find.</i></p> <p><i>Checking for understanding provides students with the opportunity to communicate their ideas with the correct academic language</i></p>	
EXPLORE	<p><u>PRIVATE PROBLEM-SOLVING TIME</u> Give students 5 - 7 minutes of private think time to begin to solve the problem individually. Circulate among the groups assessing students' understanding of the idea below.</p> <p><u>FACILITATING SMALL GROUP PROBLEM-SOLVING</u> As you circulate among the groups, press students to think about the relationship of the number of cups and the height of the stack of cups. After explaining their initial thinking you might say: What is the relationship of the number of cups with the height of their stack? What happens to the stack as you add cups? What quantities are related? How?</p>	<p><u>PRIVATE PROBLEM-SOLVING TIME</u> Make sure that the talking of other students does not interrupt students' thinking. If students begin talking, tell them that they will have time to share their thoughts in a few minutes.</p> <p><u>FACILITATING SMALL GROUP PROBLEM-SOLVING</u> The teacher's role when students are working in small groups is to circulate and listen with the goal of understanding students' ideas and asking questions that will advance student work.</p>	

The questions listed throughout this EXPLORE phase are designed to promote **INSTRUCTIONAL CONVERSATIONS**:

During the implementation of the task the students are provided with a **GRAPHIC ORGANIZER**:

As the students interact with the task they work in small **GROUPS** using **REALIA** (the cups and rulers)

Developing **ACADEMIC LANGUAGE** in context

As the teacher asks assessing and advancing questions to students while they work in small groups in the explore phase, s/he stimulates student-student discussions and models questioning.

The graphic organizer will help students to organize their thinking and connect the multiple representations of the task so that students - in particular EL/SEL - can construct their understanding of "constant rate of change", "slope" and "intercept"

A cooperative and communal learning environment is created for EL/SEL as they work in small groups, using realia to explore the task and collaboratively construct their mathematical understandings.

Teachers have an opportunity to promote students' academic language development as they facilitate the small group exploratory work, asking questions that will bring out the vocabulary being developed and assist students to make connections between the academic vocabulary and the real-world meaning/context

What do I do if students have difficulty getting started?

Allow students to work in their groups to solve the problem. Assist students/groups who are struggling to get started by prompting with questions such as:

- **What would you need to do first?**
- **How could you tell what the relationship between one cup and the stack is?**
- **What comparisons can you make?**
- **What have you tried? Have you tried fewer cups? Is there a pattern? If so, can you describe it?**
- **What can you do to test different cases?**

What misconceptions might students have?

Look for and clarify any misconceptions students may have.

What do I do if students have difficulty getting started?

By asking a question such as, "**What is the problem asking me to do? Have I solved a similar problem before?**" the teacher is providing students with a question that can be used over and over when problem-solving. This will help them focus on what they know, what they were given, and what they need to determine.

What misconceptions might students have?

Misconceptions are common.

Some strategies for helping students gain a better understanding include:

- Ask students to look for connections among the representations (the formula and the graph) of the problem.

- a. *Not understanding the constant rate of change. What is changing and what remains the same?*
- b. *Not understanding dependent/independent variables. Remind them that y represents the quantity that changes according to changes in x.*
- c. *Formulas are effective recipes for producing the dependent variable for given values of the independent variable. How is y related to x? What happens to y when x increases? What happens to y when you use different values for x?*

Which problem-solving strategies might be used by students?
How do I advance students' understanding of mathematical concepts or strategies when they are working with each strategy?

Students will represent the relationship between the cups and the stack with a formula and a graph. Questions for assessing understanding and advancing student learning are listed for each.

The formula
What is x? What is y?

POSSIBLE RESPONSES:

- a. Students might say that the height of a cup remains constant but the height of the stack changes as you add more cups.
- b. Students should realize that the height of the stack changes according to the number of cups.
- c. Students should understand that y increases as x increases.

Which problem-solving strategies might be used by students?
How do I advance students' understanding of mathematical concepts or strategies when they are working with each strategy?

POSSIBLE RESPONSES:

a.	no. of cups	height of stack
	1	4 cm
	2	4.5 cm
	3	5 cm
	4	5.5 cm
	5	6 cm
	6	6.5 cm

The formula
The formula is $y = .5x + 3.5$, where x represents the number of cups in the stack; y represents the height. Students might derive this formula by noticing in the chart that there is a difference of .5 between every two entries of height for which there is a difference of one between every two entries of "no. of cups."

How can you see the rate of change in the formula?

Does it make sense to let the value for x be negative? Why or why not?

What happens if you input 0 in the formula?

Is the function increasing or decreasing? How can you tell from the formula?

What is the rate of change in this situation?

The graph

How can you see the rate of change in the graph?

The rate of change is the coefficient of x . In this case it is $.5$, which means the height of the stack increases $.5$ cm for every cup added. This is represented by the change of $.5$ from one cup to the next in the chart.

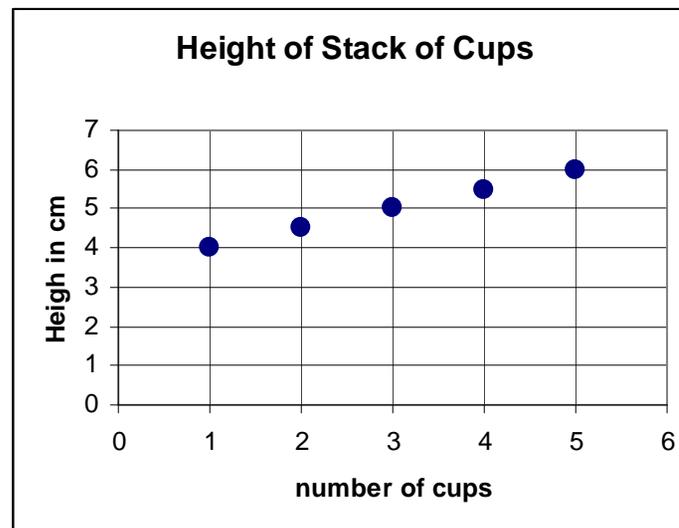
It would not make sense for x to be negative since you cannot have a negative number of cups in the stack.

If you put 0 in the formula, you get a value of 3.5. This would mean that for 0 cups the stack would be 3.5 cm high. Although this does not make sense for this problem, the 3.5 is the y -intercept of the equation.

The function is increasing since the coefficient of x is positive.

The rate of change, $.5$, is the amount the height increases for every cup added.

The graph



The rate of change can be seen by the amount of change vertically from one point to the next for every unit. In this case, the height increases $.5$ cm for every 1 cup resulting in a rate of change of $.5$ cm per cup.

	<p>At what point does the line cross the vertical axis? What do the coordinates of this point mean in the context of the Stack of Cups?</p> <p>Is the function increasing or decreasing? How can you tell from the graph?</p>	<p>The line would cross the y-axis at 3.5 cm. The coordinates of this point, the y-intercept, would mean that the height of the stack would be 3.5 cm for 0 cups.</p> <p>The function is increasing because the line is rising from left to right. As the number of cups increases, so does the height of the stack.</p>
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	Students discuss their solutions and explain their multiple representations, demonstrating understanding of the critical learnings of the lesson	
S H A R E, D I S C U S S, A N D A N A L Y Z E	<p>FACILITATING THE GROUP DISCUSSION <i>What order will I have students post solution paths so I will be able to help students make connections between the solution paths?</i> As you circulate among the groups, look for solutions that will be shared with the whole group and consider the order in which they will be shared. <i>Ask students to explain their solutions to you as you walk around. Make certain they can make sense of their solutions in terms of their representations.</i></p> <p>Ask students to post their work in the front of the classroom.</p> <p><i>What question(s) can I ask throughout the discussion that will help students keep the context and the goal of the problem in mind? (Advancing Questions)</i></p> <p>How did you use the picture to decide what the relationship between the number of cups and the stack of cups? Was the picture helpful in seeing this relationship?</p> <p>*What remained constant? What is the constant rate of change?</p> <p>*Describe the shape of your graph. Why is it a straight line? What would the line look like if this were a decreasing function?</p>	<p>FACILITATING THE GROUP DISCUSSION <i>What order will I have students post solution paths so I will be able to help students make connections between the solution paths?</i></p> <p>Even though you may display all solution paths, you should strategically pick specific solution paths to discuss with the whole group.</p> <p><i>What question(s) can I ask throughout the discussion that will help students keep the context and the goal of the problem in mind?</i></p> <p><i>Advancing questions</i> have been provided because they will help to stimulate student interest, maintain the focus of the discussion on the problem context, and focus the discussion on key mathematical ideas. Many of the questions require students to take a position or to wonder about mathematical ideas or problem-solving strategies.</p> <p><u>Accountable TalkSM</u> <i>Pointing to the graph.</i> <i>Asking student to point to the graph and showing where the evidence for their reasoning is.</i></p> <p><i>Repeating or Paraphrasing Ideas.</i> Ask other students to put explanations given by their peers into their own words. This is a means of assessing, understanding, and providing others in the class with a second opportunity to hear the explanation.</p>

The Share, Discuss and Analyze phase of the lesson provides opportunities for students to use academic language in conversations that promote analysis, reflection and critical thinking to develop a deeper understanding of the academic content. By strategically organizing solutions presented by different groups, the teacher helps students deepen their understanding of the mathematics by moving from the simplest to the most complex solution. This also provides an opportunity for students to engage in instructional conversations with the teacher and other students, thus scaffolding the learning experience for students, which is very important for English Learners and Standard English Learners. Through questioning and the resulting dialogue, students are taught the specialized language of mathematics, which is essential for EL/SEL (Constant Rate of Change, Linear Relationship, Slope, y-intercept, etc.).

Follow up Lessons:

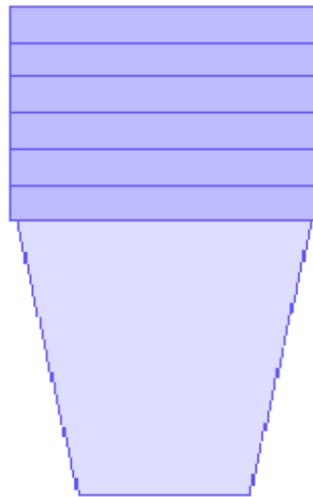
To deepen students' understanding of linear relationships and to continue to foster the development of academic language the following are suggested:

- Comparing Linear Graphs
- Linear Function Graph Matching Activity
- Sentence Builders
- Comparison Sentence and Paragraph frames
- Mathematically Speaking (Structured Instructional Conversations)

A Stack of Cups

Your task is to represent the relationship between the number of cups and the height of the stack of cups using a formula and a graph. Then answer the questions below about the stack of cups.

The paper cups shown below are identical and drawn full size:



By making measurements, represent the relationship between the number of cups and the height of the stack using a formula and a graph.

