



Fostering Connections and Using Representations in the Elementary Mathematics Classroom

*Virginia Council of Mathematics Specialist
Culpeper, VA
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Session Agenda

- VDOE Mathematics Updates and Resources
- Fostering students' ability to make mathematical connections and use effective and appropriate representations
 - What does it mean?
 - What does it look like in the classroom?
 - Role of teacher
 - Role of student
 - How do you support teachers as they strive to help students make connections and use effective and appropriate representations in the classroom?

SOL 2014 Update

NEWS RELEASE: August 27, 2014

Students Make New Gains on Mathematics SOLs

Student achievement improved during 2013-2014 on challenging mathematics Standards of Learning (SOL) tests first introduced three years ago, the Virginia Department of Education (VDOE) reported today.

| Mathematics Test | 2011-2012 | 2012-2013 | 2013-2014 |
|----------------------|-----------|-----------|-----------|
| Grade 3 Mathematics | 64 | 65 | 67 |
| Grade 4 Mathematics | 70 | 74 | 80 |
| Grade 5 Mathematics | 67 | 69 | 73 |
| Grade 6 Mathematics | 74 | 77 | 76 |
| Grade 7 Mathematics | 58 | 61 | 65 |
| Grade 8 Mathematics | 60 | 61 | 67 |
| Algebra I | 75 | 76 | 79 |
| Geometry | 74 | 76 | 77 |
| Algebra II | 69 | 76 | 82 |
| Combined Mathematics | 68 | 71 | 74 |



Computer Adaptive Testing (CAT) Update

- The Grade 6 Mathematics test will be administered in CAT format beginning in the 2014-2015 school year
- Transition of other tests to CAT is dependent on available funding; timeline is currently unknown
- Newly posted resources, including videos, can be found on the VDOE Website.



Mathematics SOL Revision Process

- Anticipated Timeline
 - January 2015 – timeline goes to the BOE
 - Spring/Summer 2015 – review committees
 - Fall 2015 – steering committee review
 - January 2016 – first review for BOE
 - Implement (2009/2016 SOL – 2016-2017)
 - Implement (2016 SOL – 2017-2018)
- Technical assistance document
- Vertical articulations revisited



VDOE Instructional Resources



VDOE Home

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STANDARDS OF LEARNING (SOL) & TESTING

Teacher *Direct*

TeacherDirect is designed to establish a direct line of communication with classroom teachers and educators.



The goal of TeacherDirect is to provide a way to share new instructional resources created by VDOE staff as well as make teachers aware of professional development and grant opportunities, and other information of special interest to teachers and their students.

[Download a Teacher Direct flier](#) to distribute or post to help get the word out.

SUBSCRIBE NOW

[Sign up](#) for weekly email notifications of updates to TeacherDirect News.



Why subscribe to TeacherDirect?

Virginia teachers discuss the benefits of subscribing to TeacherDirect and receiving weekly updates from VDOE on new SOL resources and information.



- ▶ [Play Flash Video](#) – Closed captioned and requires the free Adobe [Flash Player plug-in](#).
- ▶ [Play MP4 Video](#) – Requires the free Apple [QuickTime player plug-in](#). Right click MP4 link to save.
- ▶ [Play WMV Video](#) – PowerPoint-compatible and requires the free Windows [Media Player plug-in](#). Right click WMV link to save.

VDOE – Resources

- Standards of Learning
- Curriculum Framework
- Testing Blueprints
- SOL Practice Items and Tools Practice
- 2013 Released Test Items
- 2013 Student Performance Analysis
- Technical assistance document – sample on next slide
- Plain English Mathematics Test Information (May 2014) 
- ESS Sample Lesson Plans
- Instructional Videos 
- Vocabulary Resources – K-8, Geometry, and Algebra Cards
- SOL Institutes (2009, 2010, 2011, 2012, 2013)



Technical Assistance on Specific Content

Provides

- clarification on intent of content objectives
- instructional suggestions
- connections between content

*Based upon questions and feedback from educators
and student data analysis*

Technical Assistance on Selected SOL

Updated January 2014



Technical Assistance on Selected Content of the 2009 *Mathematics Standards of Learning*

Standard of Learning 2.17

The student will use data from experiments to construct picture graphs, pictographs, and bar graphs.

Picture graphs are graphs that use pictures to show and compare information. Pictures used in picture graphs represent one object. Pictographs are a form of picture graph that uses symbols to represent one or more objects to show and compare information. A key should be provided for the symbol in a pictograph (e.g., represents five people in a graph).

Standards of Learning 3.11, 4.9, and 5.10 – Elapsed Time

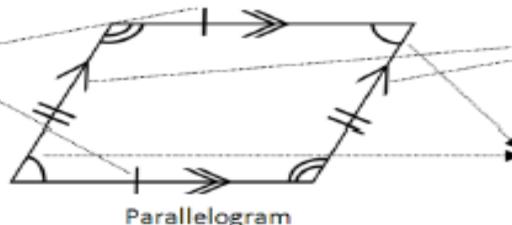
In elapsed time problems, there are three common elements: a beginning time, an ending time, and the amount of time that has elapsed. If given any two of these three elements, the students should be able to find the missing element.

- Grade 3 – Times should not cross between a.m. and p.m.
- Grade 4 – Times can cross between a.m. and p.m.

Standards of Learning 6.12 and 6.13

Students should have experiences using geometric markings in figures to indicate congruence of sides and angles and to indicate parallel sides. Geometric markings used are shown in the parallelogram.

Equal numbers of hatch marks indicate that sides are equal in length.



Equal numbers of arrows indicate that sides are parallel.

Equal numbers of angle curves indicate that angles have the same measure.



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MATHEMATICS

PROFESSIONAL DEVELOPMENT RESOURCES

VDOE provides targeted professional development through Mathematics Standards of Learning (SOL) Institutes. Since 2009, the institutes have:

- outlined the content standard changes from the 2001 Mathematics SOL to the 2009 Mathematics SOL (2009);
- supported district leaders and teachers in the implementation of the 2009 Mathematics SOL (2009, 2010, 2011, 2012);
- provided training in the vertical progression of content and pedagogy (2010);
- provided instructional guidance in content areas of greatest challenge (2010);
- provided professional development resources focused on facilitating students' mathematical understanding through problem solving, communication, and reasoning (2011); and provided professional development resources focused on the use of formative assessment resources to drive instructional decisions (2012).

The 2011 Mathematics SOL Institutes continue to support implementation of the 2009 Mathematics Standards of Learning (SOL), framed by the five goals for students becoming mathematical problem solvers, communicating mathematically, reasoning mathematically, making mathematical connections, and using mathematical representations to model and interpret practical situations.

Mathematics SOL Institutes

STANDARDS OF LEARNING

[Mathematics K-12](#)

Includes:

- Curriculum Frameworks
- Enhanced Scope & Sequence Guides
- Test Blueprints
- Released Tests and
- Practice Items

[Expand All](#) | [Collapse All](#)

2014

+ 2013 – Modifying Resources to Promote Problem Solving and Critical Thinking

+ 2012 – Using Formative Assessment Resources to Drive Instructional Decisions

+ 2011 – Facilitating Students' Mathematical Understanding through a Focus on Process Goals for Students

+ 2010 – Implementing New Content and Increased Rigor of the 2009 Mathematics Standards of Learning

+ 2009 – Overview of Changes from the 2001 to 2009 Mathematics Standards of Learning



“The content of the mathematics standards is intended to support the five goals for students”

- 2009 *Mathematics Standards of Learning*



Five goals...for students to

become mathematical **problem solvers** that

- **communicate** mathematically;
- **reason** mathematically;
- make mathematical **connections**; and
- use mathematical **representations** to model and interpret practical situations

Process Goals



2009 – Changes in Mathematics Standards

Changes to Standards of Learning (SOL) Content

- Increase rigor through an emphasis on multistep problems and application
- Move content between grade levels/subjects to improve the vertical progression
- Remove content from a grade levels/subjects
- Remove content repeated among grade levels
- Add new content
- Reduce language that limits content



2010 – Vertical Articulations and Pedagogy

| Vertical Articulation Technical Assistance Document - Grade 3 Through Grade 6 | | | |
|---|---|--|--|
| | Grade 3 | Grade 4 | |
| Counting/ Cardinality/Place Value | 3.1 a) read/write 6-digit numerals; ID place value/value of each digit; b) round whole numbers 9,999 or less to nearest 10/100/1,000; c) compare two whole numbers 0 - 9,999 w/ symbols/words | 4.1 a) ID orally/in writing place value for each digit in a whole number through millions; b) compare two whole numbers through millions w/ symbols; c) round whole numbers through millions to nearest 10/100/1,000/10,000/100,000 | 5.1 ro neare |
| Modeling/Comparing/Ordering | 3.3 a) name/write fractions rep by model; b) mode/write fraction's names; c) compare fractions w/like/unlike denominators 3.6 represent mult/div using area/set/number line models, create/solve problems involving mult of two whole numbers 99 or less and 5 or less | 4.2 a) compare and order fractions/mixed numbers; b) represent equivalent fractions; c) ID division statement that represents a fraction 4.3 a) read/write/represent/ID decimals through thousandths; b) round to whole, tenth, hundredth; c) compare/order; d) write decimal and fraction equiv from a model 4.5 a) determine common multiples/ factors | 5.2 a) equiv comp 5.18 c) using 5.3 a) comp chara |

Mathematical Problem Solving
Mathematical Communication
Mathematical Reasoning
Mathematical Connections
Mathematical Representations

MATHEMATICS

INSTRUCTIONAL VIDEOS FOR TEACHERS

These instructional videos are provided as support for the implementation of the 2009 Mathematics Standards of Learning.

- ON THIS PAGE:
- [Strategies Across the Strands](#)
 - [Number & Number Sense](#)
 - [Computation & Estimation](#)
 - [Probability & Statistics](#)
 - [Patterns, Functions & Algebra](#)
 - [Additional Resources](#)

The videos on this page link to Flash files which require the Adobe Flash Player plug-in. [Download the free player](#).

Strategies Across the Strands

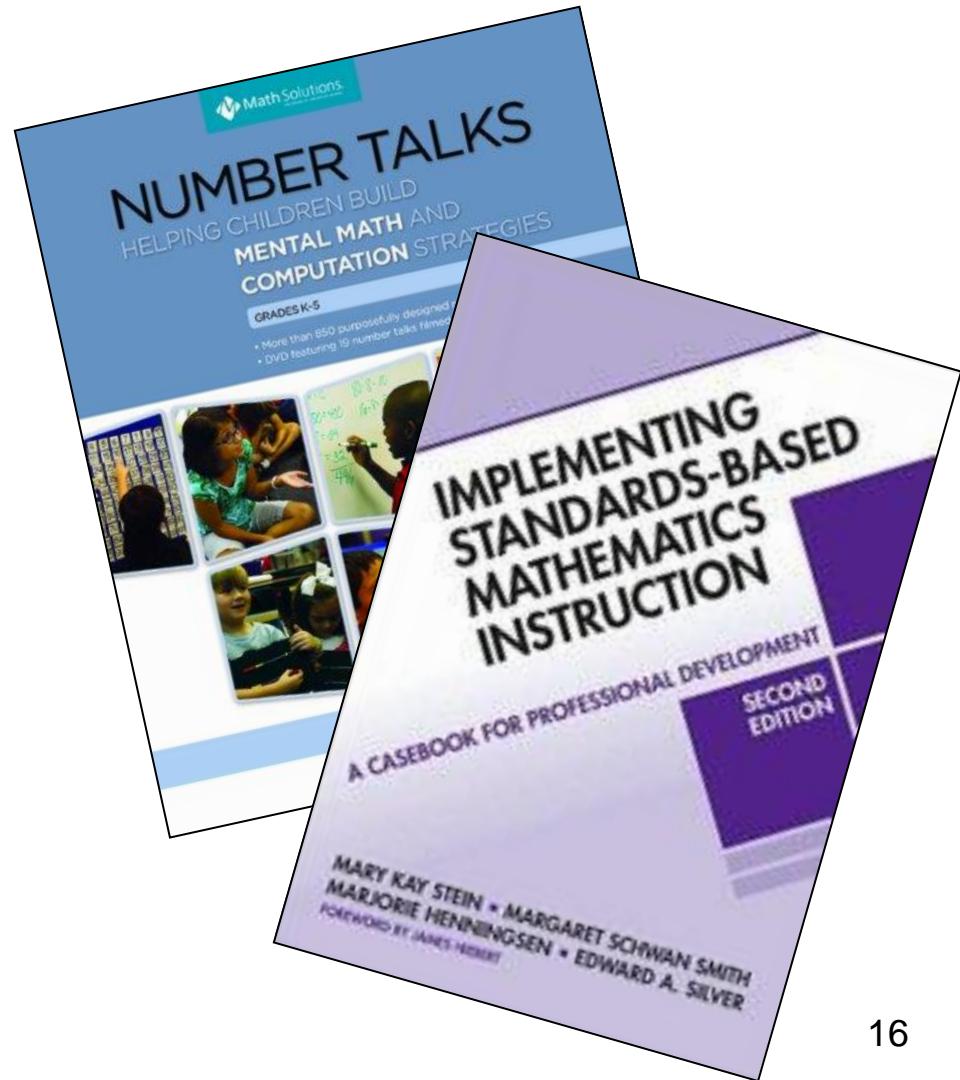
- Play Video [Working with Vocabulary / Concept Development \(grades 4-8\)](#)
Dr. Lois Williams, VDOE mathematics specialist, on the Frayer Model for vocabulary/concept development. [Handout available: Working with Vocabulary / Concept Development \(Word\)](#)
- Play Video [Vocabulary \(grades 6-8\)](#)
Dr. Lois Williams shares a technique to assist students who confuse common mathematics terms such as area, volume, and perimeter.
- Play Video [Notebooks for Organization \(grades 6-8\)](#)
Dr. Lois Williams explains an organizational system for middle school mathematics classes.
- Play Video [Multi-Step Problem Solving \(grades 4-8\)](#)
Cheryl Gray, Spotsylvania County Schools middle school mathematics specialist, on how to approach multi-step word problems. [Handout available: Multi-Step Problem Solving \(PPT\)](#)
- Play Video [Scientific Calculator Use \(grades 6-8\)](#)
Dr. Lois Williams explains the use of scientific calculators in Virginia middle schools. [Handout available: Scientific Calculator Manual \(PDF\)](#)

Number & Number Sense

- NEW Play Video [Developing Early Number Sense \(grades K-2\)](#)
Laura Domalik, Hanover County provides instructional strategies for counting and vocabulary. The strategies include identifying more than (+1), one less than (-1), basic fact concepts of +1 and -1, and missing addends.
- NEW Play Video [Using a Beaded Number Line \(grades K-2\)](#)
Laura Scearce, Hanover County, provides instructional strategies for using beaded number lines to determine backwards, rounding, adding, and subtracting.
- NEW Play Video [Modeling Equality \(grades K-2\)](#)
Debi Godfrey, Henrico County, provides activities with pan and number balances to represent and build an understanding of equality.

2011 – Cognitive Demand and Process Goals

Mathematical Problem Solving
Mathematical Communication
Mathematical Reasoning
Mathematical Connections
Mathematical Representations





2012 – Using Formative Assessment Tools

Virginia Department of Education
Fall 2012 Mathematics Standards of Learning Institutes

Facilitator's Guide
Grade Band 6-8

2012 Mathematics SOL Institutes
The purpose of the 2012 Mathematics Standards of Learning (SOL) Institutes was to provide teachers with professional development on using formative assessment resources to drive instructional decisions.

Introduction and Instructions
The product of the 2012 Mathematics SOL Institutes is a set of six online professional development modules designed to be used by a group of teachers of a specific grade level or course. Modifications could be made to adapt the professional development for more than one grade level or for large groups. Each group of teachers should select a facilitator for which this Facilitator's Guide was written. Facilitators should review the activities and handouts prior to facilitating this professional development. When scheduling for this professional development, please allow adequate time to complete all six of the modules while minimizing the amount of time between module sessions.

A Mathematics Performance Task is an assessment that requires students to synthesize mathematical content in a problem-solving setting that requires communication, reasoning, use of multiple representations, and making mathematical connections. Read more about [Virginia's Process Goals](#) for students.

| Time | Facilitator Instructions | Materials |
|------------------|---|---|
| 60 minutes total | Module 1: Performance Tasks Purpose: To anticipate common student misconceptions and errors associated with a mathematics performance task. | Handouts Needed: - Mathematics Performance Task (the Grade 7 performance task may be used by teachers of grades 6-8) - Grade 7 - Common Misconceptions and Errors Recording Sheet (a summary recording sheet will be used in Module 3) |
| 15 minutes | 1) Distribute the performance task. 2) Ask participants to work on the performance task individually. While working on the task, participants should anticipate and record common student misconceptions and errors on the Common Misconceptions and Errors Recording Sheet. | Other: - Grade 7 student video |
| 45 minutes | 1) Have participants discuss their solution strategy with a partner or small group and then the facilitator should select varied solutions to be shared with the whole group. 2) Give participants guiding questions to answer as they view the video. Show student video (6:30 min). (Facilitator may suggest looking for: independent student work; small group discussion; facilitator enhancing task through guided learning; group share of strategies) Have participants view the video keeping the following statements in mind: - Give examples of the teacher acting as a facilitator of learning (i.e., wait | |

Virginia Department of Education – Fall 2012

Mathematical Problem Solving
Mathematical Communication
Mathematical Reasoning
Mathematical Connections
Mathematical Representations

Student B

Math 7

Jenna is training for the women's four miler race in Charlottesville. She runs on a trail located in her neighborhood. According to her map, the trail is 11 inches long. The key shows that for every 1.25 inches of trail on the map, she will run ¼ of a mile. The beginning and end of the trail do not connect. A lap will require Jenna to run to the end of the trail, turnaround, and run back to the beginning.

Ratio

$\frac{25}{1.25} = \frac{x}{\frac{1}{4}}$

If she wants to run exactly 4 miles, how many more laps does she need to run? Explain exactly how much more it is. If it helps, carefully explain your reasoning and show your work.

$25 \times 4 = 100$

$1.25x = 100$

$x = 80$

Mathematics Performance Task Creation: The Process

- Content**
 - Identify the content to be assessed
 - What student content objectives will be assessed through this task?
 - What are measurable indicators of mastery?
 - What is the purpose of this performance assessment task: formative (prior to or during) or summative?
- Context**
 - Develop a meaningful and relevant context for this task that will engage students
 - What is a relevant, meaningful context for this task that will engage students?
 - What connections to instructional content could be made?
- Process Goals**
 - Identify process goals that could be highlighted in the task
 - What could students be asked to do?
 - Think critically
 - Use reasoning skills to problem solve
 - Communicate
 - Use multiple representations and models
 - Extend/copy content
 - Make connections to other mathematics
- Design**
 - Design the task
 - Sources
 - Will the task be developed from scratch?
 - Is there an existing assessment that could be modified or enhanced?
 - Construction
 - Are the student instructions clear and concise?
 - What are common misconceptions/errors?
 - Does the task address the common misconceptions/errors?
 - Is the task to be completed individually or in a group?
 - How long will the task take students to complete it?
 - Does the task allow for student choice?
- Evaluation**
 - Evaluate student work
 - What is the purpose of this performance assessment task: formative (prior to or during) or summative?
 - What criteria will be used to evaluate student work?
 - What would be an exemplary response to this task?
- Reflection**
 - Review and revise the task
 - What insights from student work will assist in revising the task?
 - What other concepts and skills might need additional instruction and assessments?

Virginia Department of Education

Fall 2012



2013 –Modifying Instructional Resources to Promote Problem Solving and Critical Thinking

Virginia Department of Education Fall 2013 Mathematics Standards of Learning Institutes

2013 Mathematics SOL Institutes

The purpose of the 2013 Mathematics SOL Institutes is to provide teachers with professional development focused on the relationship between curriculum, instruction, and assessment, by targeting the processes of analysis and modification of existing resources to match student learning expectations and promote problem solving.

Introduction and Instructions

This grade-band professional development will be comprised of two components:

- Module 1 Parts 1-4: Analyzing and modifying assessments – Participants will compare expectations of SOL and Curriculum Framework to an assessment and modify it to meet intended expectations.
- Module 2 Parts 1-3: Modifying mathematical tasks to promote problem solving – Participants will modify existing mathematical tasks to emphasize the use of process skills and problem solving.

The product of the 2013 Mathematics SOL Institutes is a set of online professional development modules designed to be used by a group of teachers of a specific grade level or course. Modifications could be made to adapt the professional development for more than one grade level/course or for large groups. Each group of teachers should select a facilitator for which this Facilitator's Guide was written. Facilitators should review the activities and handouts prior to facilitating this professional development.

| Approximate Time | Facilitator Instructions | Materials |
|------------------|---|---|
| 30 minutes total | Module 1 Part 1: Analysis of Assessments <ol style="list-style-type: none">Select and distribute the appropriate assessment for your whole group.Ask participants to complete the assessment individually. While working, participants should be thinking about whether or not it is a "good" assessment and why.Have participants discuss their conclusions with a partner or small group.Have small groups share and record their conclusions and justifications with the whole group.View the Assessment Analysis Guiding Questions document and discuss similarities | |
| 15 minutes | | <ul style="list-style-type: none">Mathematics Assessment (select the assessment for your course)<ul style="list-style-type: none">Grade 3Grade 4Grade 5Assessment Analysis Guiding Questions |

- Mathematical Problem Solving**
- Mathematical Communication**
- Mathematical Reasoning**
- Mathematical Connections**
- Mathematical Representations**

Assessment Analysis Guiding Questions

SOL ALIGNMENT

- Does the assessment assess the standard and targeted components of the Essential Knowledge and Skills in the Curriculum Framework?
- Does the assessment reflect the requirements of the verbs found in the Curriculum Framework?
- Did instruction go beyond the standard?
 - If so, does the assessment reflect your instruction?
 - If not, does the assessment adhere to the parameters of the SOL?

LEVEL OF COGNITIVE DEMAND

- Does the assessment have an appropriate variety of questions requiring various levels of cognitive demand?
- Does the assessment require students to explain and justify?

FORMAT

- Does the assessment provide various ways for students to demonstrate understanding (open response, multiple choice, etc.)?



2014 – Making Connections and Using Representations

- The purpose of the 2014 Mathematics SOL Institutes is to provide teachers with professional development focused on the instruction that supports process goals for students in mathematics.
- Particular emphasis will be on **fostering students' ability to make mathematical connections** and **use effective and appropriate representations** in mathematics.



Session Agenda

Fostering students' ability to make mathematical connections and use effective and appropriate representations

- What does it mean?
- What does it look like in the classroom?
 - Role of teacher
 - Role of student
 - Role of mathematics specialist/teacher leader
- How do you support teachers as they strive to help students make connections and use effective and appropriate representations in the classroom?



Defining Connections and Representations

- What does it mean to **make mathematical connections** and **use effective and appropriate representations?**
- What words come to mind when you think about ...
 - making connections?*
 - using effective and appropriate representations?*
- *Turn-n-Talk*



Mathematical Connections

Students will **relate concepts and procedures** from different topics in mathematics to one another and see mathematics as an integrated field of study.

Through the application of content and process skills, students will **make connections between different areas of mathematics and between mathematics and other disciplines, especially science**. Science and mathematics teachers and curriculum writers are encouraged to develop mathematics and science curricula that reinforce each other.



Making Connections in Elementary School

Teaching Elementary Students the
Magic of Math



The Value of Making Connections in the Mathematics Classroom

An emphasis on mathematical connections helps students build a disposition to use connections in solving mathematical problems, rather than see mathematics as a set of disconnected, isolated concepts and skills.

National Council of Teachers of Mathematics. 2000.
Principals and Standards for School Mathematics. Reston, VA.



Making Connections

Teachers are asking questions such as:

"How is our work today with similar triangles related to the discussion we had last week about scale drawings?"

Students should routinely ask themselves,

"How is this problem or mathematical topic like things I have studied before?"

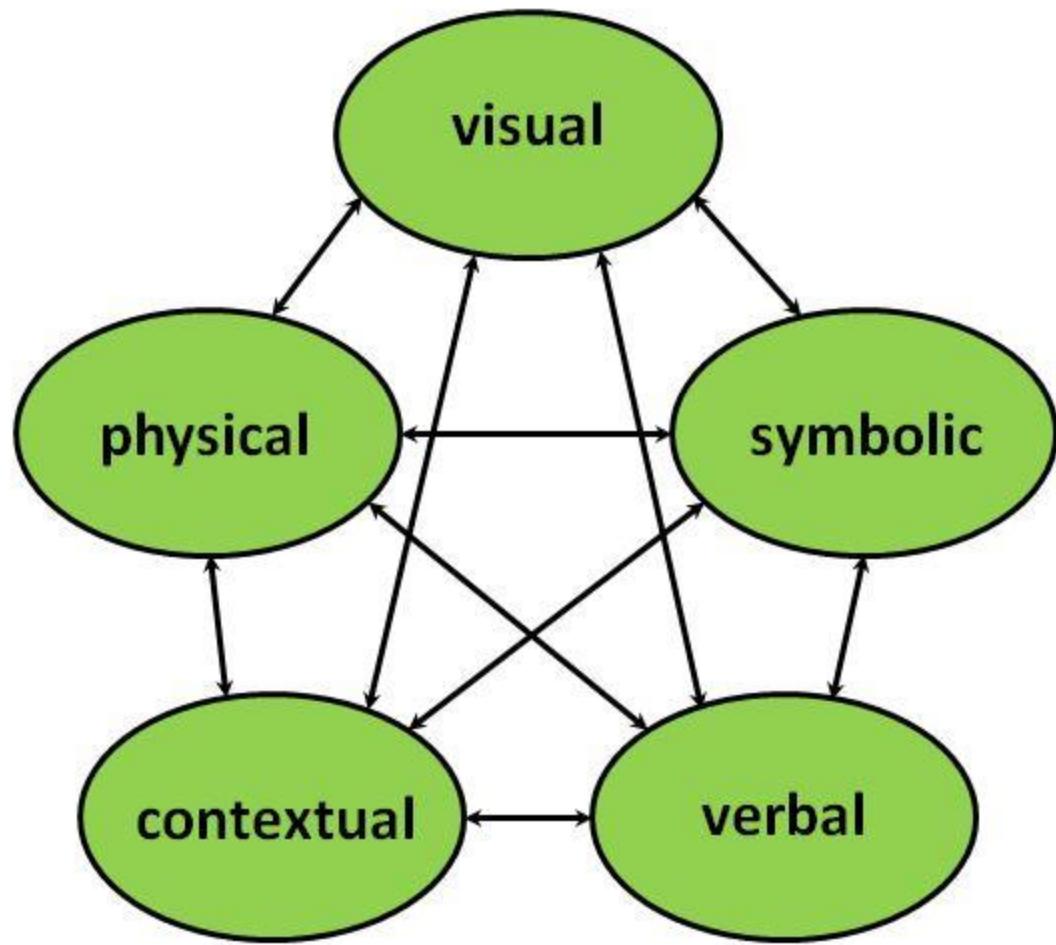


Mathematical Representations

Students will represent and describe mathematical ideas, generalizations, and relationships with a variety of methods. Students will understand that representations of mathematical ideas are an essential part of learning, doing, and communicating mathematics. Students should move easily among different representations – graphical, numerical, algebraic, verbal, and physical – and recognize that representation is both a *process and a product*.



Five Types of Mathematical Representations



Source: The National Council of Teachers of Mathematics. (2014).

Principles to Actions Ensuring Mathematical Success for All, Reston, VA: NCTM.



“Representations are useful in all areas of mathematics because they help us **develop, share, and preserve** our mathematical thoughts.

They help to **portray, clarify, or extend a mathematical idea** by focusing on its essential features.”

National Council of Teachers of Mathematics. 2000, p. 206.
Principals and Standards for School Mathematics. Reston, VA.



The Role of the Teacher

- *Create* a learning environment that encourages and supports the use of multiple representations (look for ways to connect student-created representations to more conventional representations)
- *Model* the use of a variety of representations
- *Orchestrate* discussions where students share their representations and thinking
- *Support* students in making connections among multiple representations, to other math content and to real world contexts

Adapted from: Van de Walle, J.A., Karp, K.S., Lovin, L.H. and Bay-Williams, J.M. (2013). *Teaching Student-Centered Mathematics: Developmentally Appropriate Instruction for Grades 3-5 (Volume II) (2nd Edition)*. Pearson.



Role of the Student

- *Create* and *use* representations to organize, record, and communicate mathematical ideas
- *Select, apply*, and *translate* among mathematical representations to solve problems
- *Use* representations to model and interpret physical, social, and mathematical phenomena



Taking a Look INSIDE the Classroom- Video

Looking for Representations and Connections in the Classroom- Video Reflection Handout

- First/Second Grade Combination Class
- Teacher – Tim Sears (14 year veteran)
- Probability and Animal Habitats

Video: Annenburg Learner: Teaching Math – Grades K-2 Applying Representations



Students must be **actively engaged** in developing, interpreting, and critiquing a variety of representations.

This type of work will lead to better understanding and effective, appropriate use of representation as a mathematical tool.

National Council of Teachers of Mathematics. 2000, p. 206. *Principals and Standards for School Mathematics*. Reston, VA.



Quality instruction allows all students to:

- recognize and use connections among mathematical ideas;
- understand how mathematical ideas interconnect and build on one another to produce a coherent whole; and
- recognize and apply mathematics in contexts outside of mathematics.

National Council of Teachers of Mathematics. 2000. *Principals and Standards for School Mathematics*. Reston, VA.



Role of the Mathematics Teacher Leader

How might you support teachers as they strive to help students make **connections** and **use effective and appropriate representations** in the classroom?



Mathematics Classroom LOOK FORS

- Are multiple representations and connections encouraged within the classroom? this lesson?
- What types of representations and/or connections are present in this activity?
- How are students creating and using representations to communicate their mathematical ideas?
- How is the classroom discussion enhanced by students' representations and/or connections?
- Who is doing the thinking? students or teachers?
- In what ways might teacher's purposefully plan for the use of connections and representations?



Purposefully Planning for the Use of Representations

Teachers must ask themselves,

**“What models or materials
(representations) will help convey the
mathematical focus of today’s lesson?”**

- Skip Fennell, Past-President NCTM

http://www.nctm.org/uploadedFiles/About_NCTM/President/2006_09pres.pdf



Making Connections and Using Representations - Resources

Planning Mathematics Instruction: Essential Questions

Determining Mathematical Objective(s) for Students

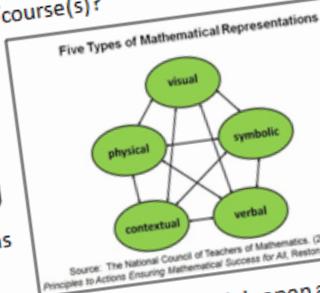
- What should students know, understand, and demonstrate?
- Which bullets from the Essential Knowledge and Skills will be addressed?
- What criteria will be used to determine student understanding?

Connecting to Prior Knowledge and Future Content

- What is the related content (prior knowledge) from previous grade(s)/course(s)?
- What is the related content in future grade(s)/course(s)?
- What representations/strategies were used to develop prior knowledge?
- How is the objective connected to the related content and to the real world?

Developing and Reinforcing Content

- What are students' common misconceptions and how will they be addressed?
- What vocabulary is essential?
- Which representations/strategies will model the mathematics and deepen and extend students' mathematical understanding?
 - What are the strengths and limitations of the representation/strategy?
 - How will the selected representation assist in student understanding?
- In what order will the content and different representations be introduced?
- What mathematical tasks will be used to develop content?
 - How is the task connected to the objective and representation/strategy?
 - Does the task require an appropriate level of cognitive demand?
 - In what ways could the task be extended to connect to other math concepts or other disciplines?
- How will students communicate their understanding?
- What differentiation will address the learning needs of students who are language learners, and gifted students?
- How will student understanding be monitored throughout instruction and assessment?



Mathematics Instructional Connections for Physical and Visual Representations

Content Connections

- Count and skip count
- Place value
- Represent fractions (length model), decimals, and integers
- Compare, order, and operate with whole numbers, fractions, decimals, and integers
- Measurement
- Represent data (line plots, balance point/mean)
- Probability
- Represent absolute value

Challenges or Limitations could include:

- Modeling multiplication versus division (motion required to build understanding)
- Modeling division by negative integers
- Modeling with number 'paths' most appropriate for PreK-Grade 1

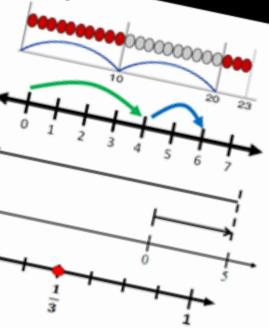
- Sort and classify geometric figures (attributes)
- Compose and decompose geometric figures
- Patterns
- Represent fractions (area model)
- Equivalent fractions/Compare fractions
- Operations with fractions
- Ratios

Challenges or Limitations could include:

- Modeling certain fractions
- One-to-one correspondence
- Count and skip count
- Compose and decompose numbers
- Operations with whole numbers
- Ratios and fractions (set or area model)
- Probability
- Measurement

- One-to-one correspondence
- Compose and decompose numbers
- Represent integers
- Properties of integers
- Operations with whole numbers
- Probability

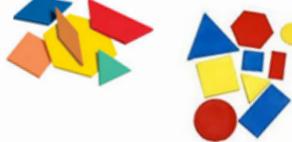
Number Lines



Bar Model



Pattern/Attribute Blocks



Square Tiles



Two-color Counters





Examining Student Work – Possible Protocol

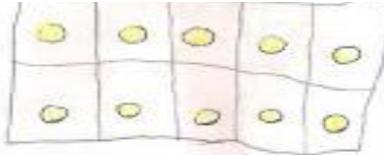
Key Questions

- What types of representations are evident in the student work?
- What does this tell you about their understanding of the concepts being developed in this lesson?
- What does the work tell you about misunderstandings or misconceptions they have?
- What are the next instructional steps needed?
- Do you see evidence students are making connections within their own work?
- How could the you use the student work to help all students make connections? (between representations, between strategies, and/or between mathematical ideas)

Grade 1 - Frogs in the Pond

There were 12 frogs at the pond. Some were swimming and some were sunning themselves on a log. There were more frogs swimming than sunning. How many frogs were swimming and how many were sunning? Use pictures, words and numbers to prove that your answer makes sense. Can you find more than one way to do this?

Grade 1



• •

$$10 + 2 = 12$$

10 swimming

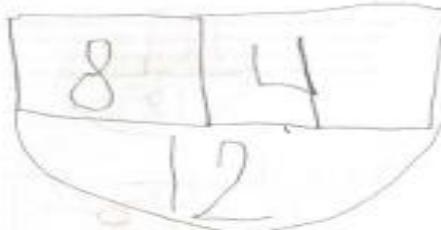
2 sunning

Student B p. 3 4-12-11

$$7 + 5 = 12$$

7 Swimming
5 Sunning

7 5



$$8 + 4 = 12$$

8 Swimming

4 Sunning

4-12-11 Student B p.4



$$11 + 1 = 12$$

11 1
12
1 2 3 4 5 6 7 8 9 10 11 12

11 Swimming

1 Sunning



During one week, Celeste read 5 books and her sister Adele read 3 books.

Grade 3

- $\frac{1}{4}$ of the books Celeste and Adele read were nonfiction.
- The rest of the books Celeste and Adele read were fiction.

1) How many non-fiction books did the sisters read last week?

2) How many fiction books did the sisters read last week?

→ 6 Fiction books

Be sure to explain your work using pictures, words, and symbols.



It is 2 nonfiction books and its
6 fiction books

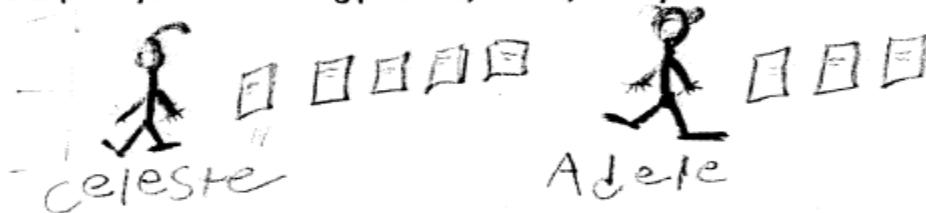


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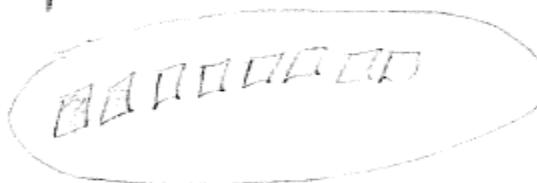
Be sure to explain your work using pictures, words, and symbols.



2 books were nonfiction and 8
books were fiction

$$\text{?} = \frac{1}{4} \text{ of } 8 \text{ is } 2 \text{ books}$$

Source: VDOE 2012 Mathematics SOL
Institute - Using Formative Assessment
Resources to Drive Instructional Decisions
(Grade 3-5)



$$\frac{2}{8}$$





Fostering Connections and Representations in Instruction

- Use data to determine areas of weakness (students and teachers)
- Be knowledgeable about and encourage the use of available resources
- Utilize PLCs and professional learning sessions to explore resources
 - Process goals for students
 - Vertical articulation of content
 - Instructional strategies
 - Development of quality assessments
 - Effective use of data
- Support teachers in purposefully planning for the use of representations and making connections in their mathematics lessons



Five goals...for students to

become mathematical **problem solvers** that

- **communicate** mathematically;
- **reason** mathematically;
- make mathematical **connections**; and
- use mathematical **representations** to model and interpret practical situations

Process Goals



QUESTIONS?

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