VACMS 2014 Conference

Advocating Appropriate Applications

Virginia Mathematics Specialists

Modifying Textbook Tasks to Increase Rigor

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Essential Questions

- *What is the criteria for a worthwhile problem or task?*
- *What are helpful strategies for modifying textbook tasks in order to increase student reasoning or thinking and there by increase the rigor?*
- *What supports do teachers need as they gain confidence and abilities to implement tasks that require students to engage in more reasoning and communication?*

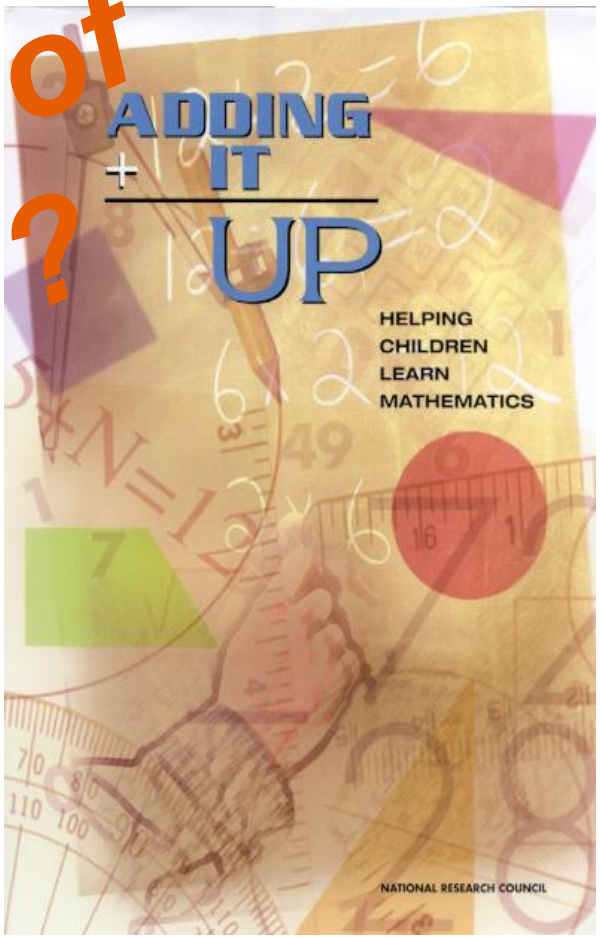
So, you want me to do WHAT and teach the curriculum!

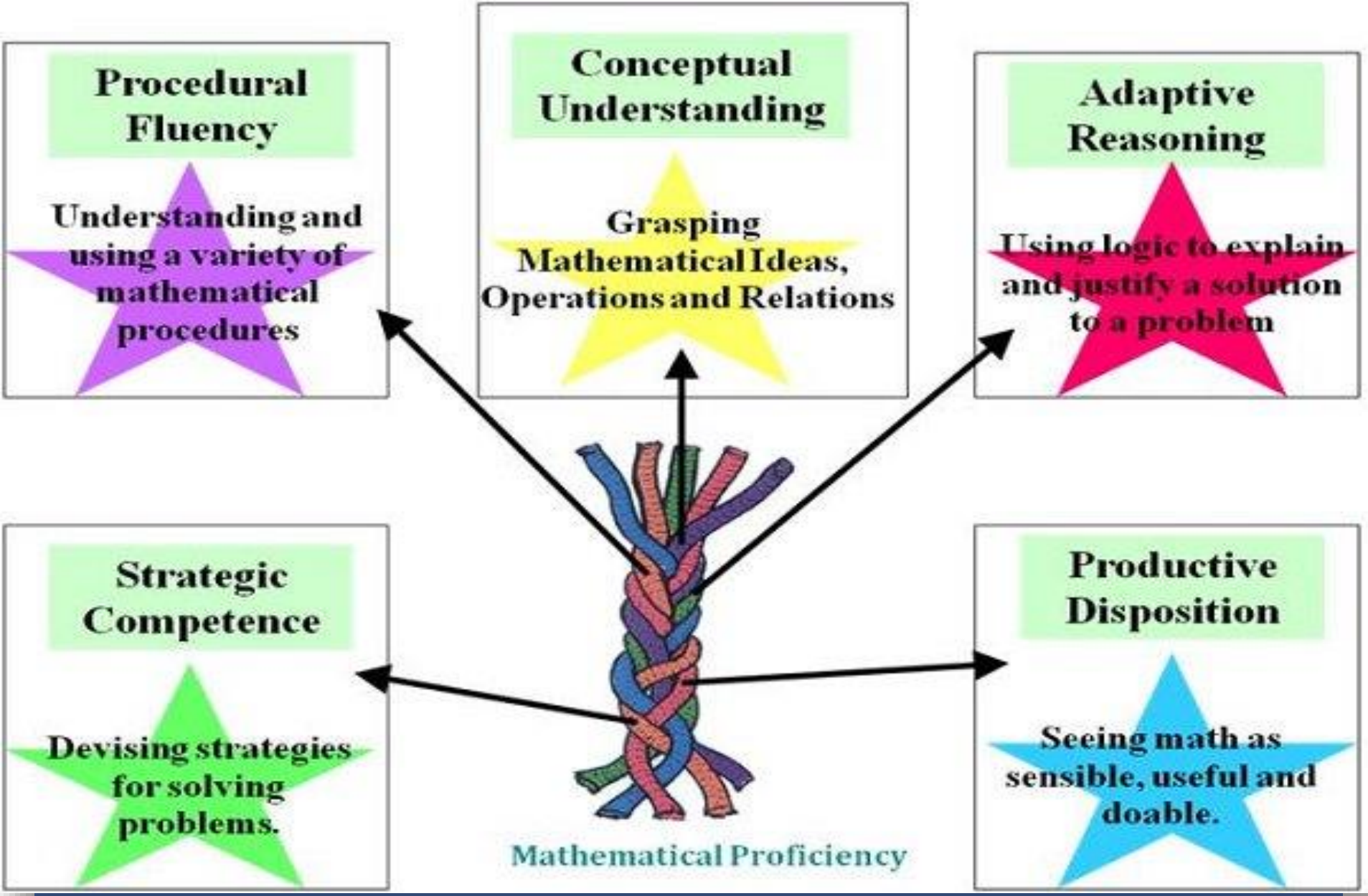
So, what can we help teachers do within the required curriculum and with the school division's adopted materials that will help them "teach up" and increase student thinking?

Synthesis of Research on Teaching Learning

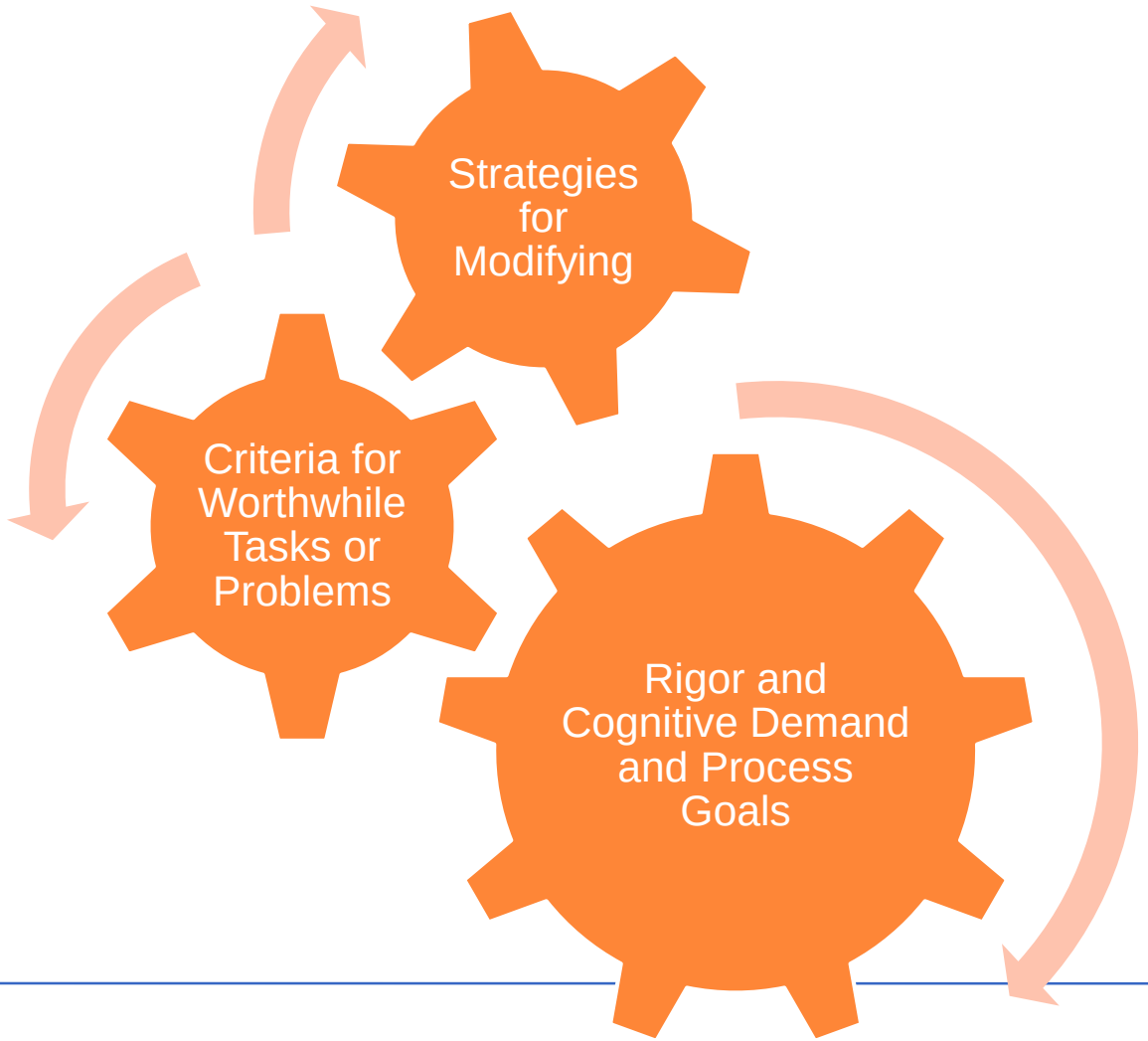
- ***Adding It Up: Helping Children Learn Mathematics.***
- National Academy Press, 2101 Constitution Ave. NW, Washington, DC 20055, or online at www.nap.edu

What is the goal of "teaching up"?




Procedural Fluency
Understanding and using a variety of mathematical procedures
Conceptual Understanding
Grasping Mathematical Ideas, Operations and Relations
Adaptive Reasoning
Using logic to explain and justify a solution to a problem
Strategic Competence
Devising strategies for solving problems.
Productive Disposition
Seeing math as sensible, useful and doable.
Mathematical Proficiency

Transition to Teaching for Mathematical Proficiency



Teaching Up Rigor in the Classroom

- **Active**, either through conversation or hands-on or minds-on activity. There's questioning and discovery going on.

- Deep rather than broad; project-based. The **learners are digging into** a topic or project.

- **Engaging.** The learner has made a real connection with the material to be learned, either on his or her own or with the help of the teacher. There's a sense that the learning was "hard but satisfying."

Depth of Knowledge and Cognitive Demand

Level 1 - Recall

- Requires students to recite or recall of information including fact, formula, or simple procedure.
- May be asked to demonstrate a rote response, use a well-known formula, follow a set procedure (like a recipe), or perform a clearly defined series of steps.

Level 2 – Skills and Concepts

- Requires some mental processing beyond recalling.
- Requires students to make some decisions on how to approach a task or problem

Level 3 – Strategic Thinking

- Requires reasoning, planning, using evidence and in most cases to "explain their thinking."
- Requires students to go beyond; to explain, to generalize, or connect ideas

Level 4 – Extended Thinking

- Requires some complex reasoning, planning, developing and thinking over an extended period of time.
- May be asked to develop a hypothesis and perform complex analysis

Compare Original and Modified Tasks

- 3 tasks
- Work in table pairs–
  - One pair evaluate for the even criteria descriptors and
  - The other pair evaluate for the odd criteria descriptors
- Use note taking sheet to record ideas
- Discuss as table group, the tasks and the modifications.

Comparing Two Versions of a Task

1. Compare each task to its modified version (A to A’, B to B’, C to C’)

2. Determine how each pair of task is the same and how it is different

3. Determine what the modifications in the task was trying to accomplish and whether the difference between a task and its adaptation matter?

Comparing Two Versions of a Task:
How are they the same and how are they different?

TASK A

MAKING COJECTURES Complete the conjecture based on the pattern you observe in the specific cases.

29. Conjecture: The sum of any two odd numbers is _____?

1 + 1 = 2	7 + 11 = 18
1 + 3 = 4	13 + 19 = 32
3 + 5 = 8	201 + 305 = 506

30. Conjecture: The product of any two odd numbers is ____?

1 x 1 = 1	7 x 11 = 77
1 x 3 = 3	13 x 19 = 247
3 x 5 = 15	201 x 305 = 61,305

TASK A'

For problems 29 and 30, complete the conjecture based on the pattern you observe in the examples.

Then explain why the conjecture is always true <u>or</u> show a case in which it is not true.

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Comparing Two Versions of a Task:
How are they the same and how are they different?

TASK B

a. Simplify each expression.

$(-2)^2$ $(-2)^3$ $(-2)^4$ $(-2)^5$

$(-3)^2$ $(-3)^3$ $(-3)^4$ $(-3)^5$

b. *Make a Conjecture* Do you think a negative number raised to an even power will be positive or negative? Explain.

c. Do you think a negative number raised to an odd power will be positive or negative? Explain.

Task B'

1. Solve the following examples.

$(-2)^2$ $(-2)^3$ $(-2)^4$ $(-2)^5$

$(-3)^2$ $(-3)^3$ $(-3)^4$ $(-3)^5$

2. Make some observations about any patterns that you notice.

3a. Using what you notice about the examples above, make a conjecture about negative numbers to an even power.

3b. How do you know that this will be true for all negative numbers?

4a. Using what you notice about the examples above, make a conjecture about negative numbers to an odd power.

4b. How do you know that this will be true for all negative numbers?

Comparing Two Versions of a Task:
How are they the same and how are the different?

Task BB

Guided Practice

Use the answer from the first problem to answer the second problem.

Alex bought 16 red apples. He bought 12 yellow apples. How many apples did he buy in all?

28 apples

Alex uses 14 of the apples. How many apples are left.

_____apples

Task BB'

Solve the problem in two different ways. Write two equations showing the two different approaches that can be taken when solving this problem.

Alex bought 16 red apples. He bought 12 yellow apples. How many apples did he buy in all?

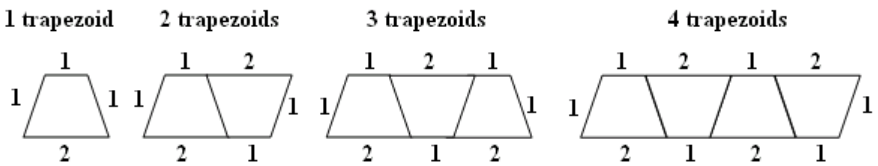
Alex uses 14 of the apples. How many apples are left?

Compare the two different ways of solving the problem and explain how the two ways differ from each other. Explain what remains the same with each way of solving the problem.

Comparing Two Versions of a Task:
How are they the same and how are the different?

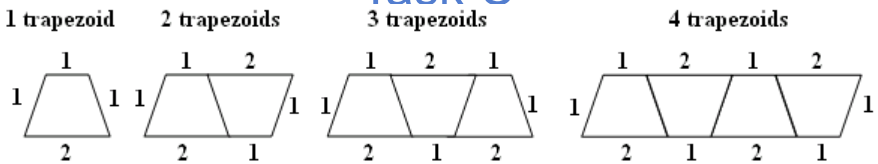
Task C

For Exercises 45 and 46, use the diagram below that shows the perimeter of the pattern consisting of trapezoids.



45. Write a formula that can be used to find the perimeter of a pattern containing *n* trapezoids.
46. What is the perimeter of the pattern containing 12 trapezoids?

Task C'



1. Make as many observations as you can about the trapezoid pattern.
2. Find the perimeter of the first four trapezoids shown above.
3. Find the perimeter of the pattern containing 12 trapezoids without drawing a picture.
4. Write a generalization that can be used to find the perimeter of a pattern containing any number of trapezoids.
5. Using words, numbers, and/or connections to the visual diagram, prove that the generalization you created in part 4 will always works.

Strategies for Modifying Tasks

Look across the three sets of tasks and consider:

- what the modifications in the tasks were trying to accomplish?
- what modification principles can be generalized?

Task Modification Strategies

1) Adding the requirement that students make observations before they engage in proving activities.

2) Adding the requirement that students explore or identify a pattern from given data or generate examples in order to search for patterns.

3) Adding the requirement that students make or revise conjectures.

4) Adding the requirement that students provide a mathematical argument or proof.

5) Removing or reducing scaffolding from a conjecture-related task so that the task is open to different approaches and does not tell exactly what to do or how to do it.

Task Modification Strategies

6) Adding the requirement that students communicate an argument using a different representation(s) (pictorial, flow chart, 2-column, paragraph) than the way the argument was initially presented, and/or make comparisons between different arguments.

7) Requiring that information be represented another way (a picture, in a table, an equation, with a context, etc.) to write about the math concept from the new representation.

8) Add a requirement to move from specific numbers to the general case; to form a generalization and to apply the generalization to other cases.

9) Require multiple solution paths and then ask for a comparison of the solution paths or mathematical relationships.

Powerful Modification Tool: *Reasoning and Justifying*

- Provide *more and different opportunities to reason and prove* than the original task.
- Create a *low threshold-high ceiling tasks* that give multiple entry points.
- Engage students in *investigation and conjecture* instead of just giving the answer.
- Generate proof (or proof-like) *arguments* without needing to use the term proof.

Mike Steele, Fran Arbaugh, and Cynthia Taylor with Peg Smith
NSF Project, Enhancing the Reasoning-and-Proving Content in Textbook Tasks

Increasing Cognitive Demand and Rigor 2009 Virginia Mathematics SOL

The content of the mathematics standards is intended to support the following five process goals for students:

- becoming mathematical **problem solvers**,
- **communicating** mathematically,
- **reasoning** mathematically,
- making mathematical **connections**, and
- using mathematical **representations** to model and interpret practical situations.

2009 Virginia Standards of Learning for Mathematics, Page iv.

NCTM 2010 Research Brief
Criteria for a worthwhile problem--

1. Important, useful mathematics embedded.
2. Requires high-level thinking and problem solving.
3. Contributes to conceptual development of students.
4. Creates opportunity for teacher to assess student learning.

All 4 Must be present

5. Approached in multiple ways using different solutions strategies.
6. Various solutions or allows different decisions or positions to be taken and defended.
7. Encourages engagement and discourse.
8. Connects to other important mathematics ideas.
9. Promotes skillful use of mathematics.
10. Opportunity to practice important skills.

Why Is Teaching With Problem Solving Important to Student Learning? 2010 NCTM Research Brief

Characteristics of a Worthwhile Problem and the Process Goals

1. Important, useful mathematics embedded. *(Content Standards)*
2. Requires high-level thinking and problem solving. *(PG Problem Solver)*
3. Contributes to conceptual development of students. *(Relational Understanding)*
4. Creates opportunity for teacher to assess student learning. *(Formative Assessment)*
5. Approached in multiple ways using different solutions strategies. *(PG Multiple Representations)*
6. Various solutions or allows different decisions or positions to be taken and defended. *(PG Reasoning)*
7. Encourages engagement and discourse. *(PG Communicating)*
8. Connects to other important mathematics ideas. *(PG Connections)*
9. Promotes skillful use of mathematics. *(PG Problem Solver)*
10. Opportunity to practice important skills. *(Procedural Fluency)*

Time for You to Try Your Hand

- Work in pairs and select one of the task on your table to modify.
- Tape the original textbook task (T) to a piece of chart paper
- Write out the modification as task T’ on the chart paper
- Post the team’s chart paper

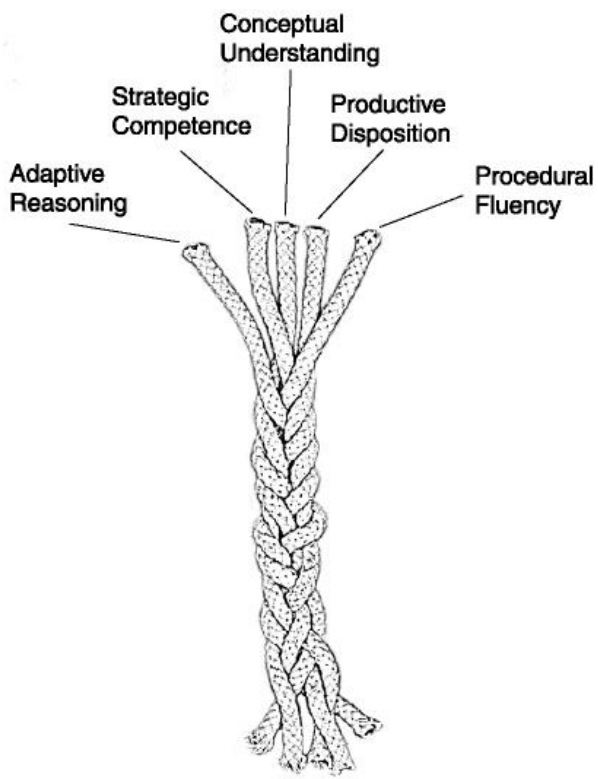
Gallery Walk and Group Reflection

- Circulate, analyzing the other teams' modified tasks.
- On sticky note describe ways in which the tasks were modified and the benefit to students.
- If the modifications do not increase the cognitive demand of the task, then ask a "wondering about" a task might be modified.

Linking Back

How is the work we have done today related to a Mathematics Teacher Leader or a Mathematics Specialist?

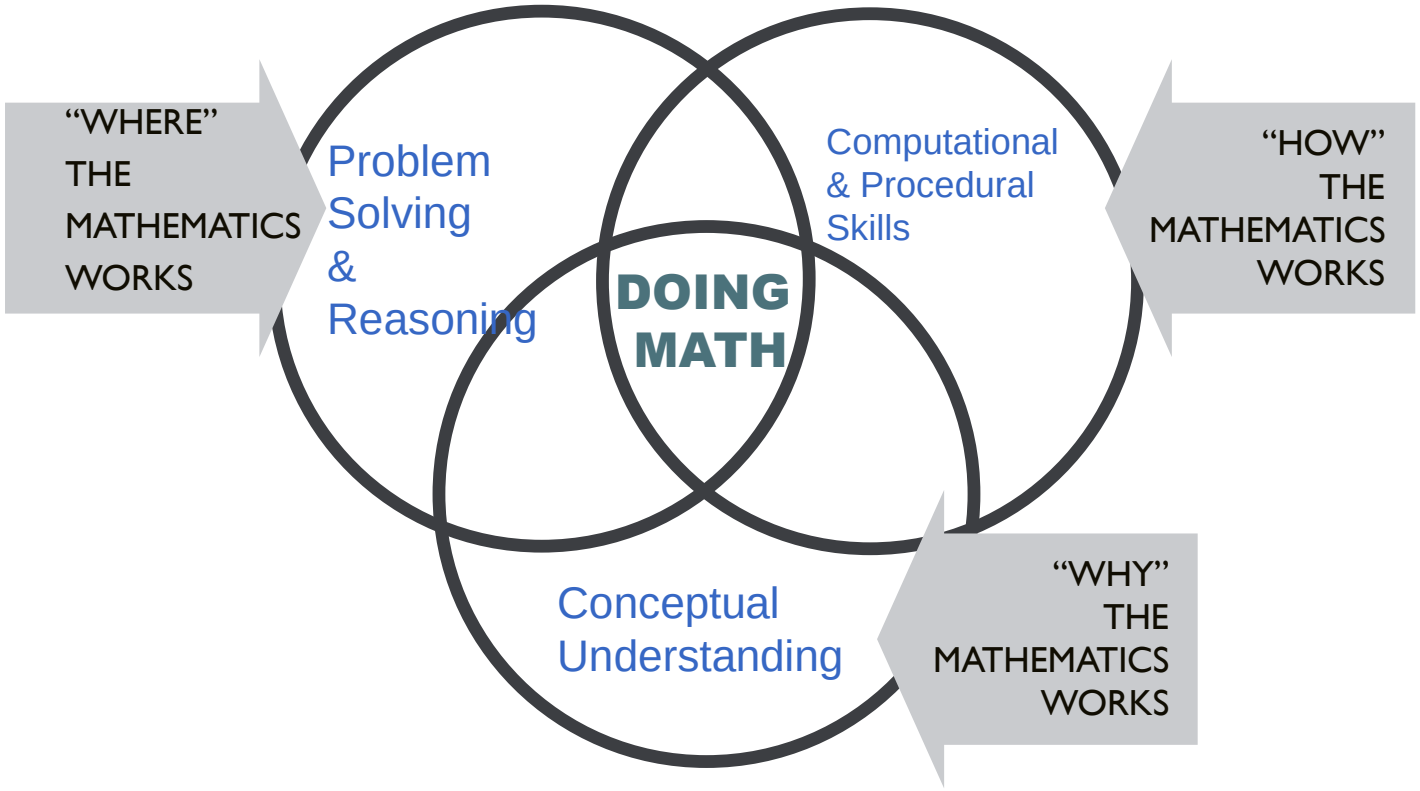
Mathematical Proficiency



Must get beyond **skills only** focus and work toward also developing **reasoning** and **understanding** in order to cultivate a **productive disposition**.

Adding It Up: Helping Children Learn Mathematics, NRC (2001)

A Balanced Mathematics Program



Teacher and Student Roles in Classroom Discourse	
Teacher's Role	**Student's Role**
Poses questions and tasks that elicit, engage, and challenge each student's thinking.	Listen to, respond to, and question the teacher and one another.
Listens carefully to student's ideas.	Use a variety of tools to reason, make connections, solve problems, and communicate.
Asks students to clarify and justify their ideas orally and in writing.	Initiate problems and questions.
Decides which of the ideas students bring up to pursue in depth.	Make conjectures and present problems.
Decides when and how to attach math notation or language to students' ideas.	Explore examples and counterexamples to investigate conjectures.
Decide when to provide information, when to clarify an issue, when to model, when to lead, and when to let different students struggle with a problem.	Try to convince themselves and one another of the validity of particular representations, solutions, conjectures, and answers.
Monitors student participation in discussions and decides when and how to encourage each student to participate.	Rely on mathematical evidence and argument to determine validity.

Source: Adapted from information in Professional Standards for Teaching Mathematics, by the National Council of Teachers of Mathematics, 1991, Reston, VA; Author.Kenney, Hancewicz, Heuer, Metsisto, Tuttle(2005).

Change is Not Easy or Comfortable

Change is Not Easy or Comfortable

Support—Support--Support

OR

Leadership III Student Reflection Ah-ha!

"Teacher progress should be made between the lesson and students progress should be made during the lesson."

Mathematics Specialist 2012

VDOE Professional Development Resources

- **2013- Online professional development modules provided on the VDOE Mathematics Web site**
  - 2 modules with 7 parts total, broken into (roughly) 45-min segments
  - Facilitators guide, all necessary documents
- **Options for delivery**
  - grade-level/subject area teams to work through professional development in their learning community meetings
  - division-wide professional development

http://www.doe.virginia.gov/instruction/mathematics/professional_development/index.shtml

Supplementary Ideas

Dan Meyer

One teachers story about modifying textbook tasks to increase student reasoning and patient problem solving.

https://www.youtube.com/watch?v=NWUFjb8w9Ps

Math Class Needs a Makeover

"So 90 percent of what I do with my five hours of prep time per week is to take fairly compelling elements of problems …from my textbook and ***rebuild them in a way that supports math reasoning and patient problem solving****."*

Dan Meyer (March, 2010)

http://blog.mrmeyer.com/-- Provides commentary on a textbook problem he modified.

Let's Go!
Complete these two puzzles

X	2a	5
3a		
-4		

X		
	$3a^2$	$4a$
	$-15a$	-20

Which caused more thinking?

X	**a**	**5**
a		
-4		

X		
	$3a^2$	$4a$
	$-15a$	-20

Let's Go!
Complete these two puzzles.

+	4	6
3		
7		

+		
	4	10
	7	13

Which caused more thinking?

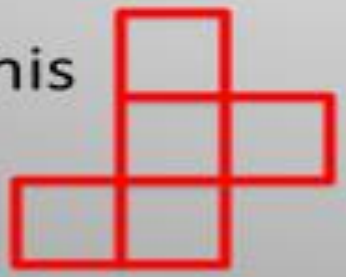
+	4	6
3		
7		

+		
	4	10
	7	13

Creating More Opportunity to Think and Problem Solve

Ask the Answer!

I have an area of $24cm^2$. What does the shape look like?

This  is 1 fifth. What does the whole shape look like?