

A Progression for Teaching Proportional Reasoning

VACMS

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How far will you travel if...

- [YouTube Video](#)



❖ It is estimated that more than half of the adult population cannot be viewed as proportional thinkers. That means that we do not acquire the habits and skills of proportional reasoning simply by getting older.

❖ Proportional reasoning is regarded as the “cornerstone of elementary mathematics and the gateway to higher mathematics.”

Lamon (1999)

GOALS

- Reflect on the progression of instructional tasks necessary for proportional reasoning
- Experience hands-on reasoning about ratio relationships
- Consider whether our teachers are *really* reasoning about proportions with their students

VA SOL for Proportional Reasoning

- 6.1- Ratios (including appropriate notations)
- 6.2- Investigating fractions, decimals, & percents
- 6.9- U.S. Customary and Metric conversions
- 6.10- Ratio of circumference of a circle to diameter

- 7.4- Practical problems with proportional reasoning
- 7.6- Similar Figures
- 7.8- Dilations

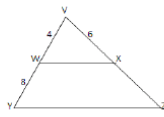
- 8.3- Practical problems with rational numbers, percents, ratios, & proportions (including percent increase/decrease)
- 8.8- Dilations
- 8.16- Graphing proportional linear equations

- A.8- Direct and inverse variation

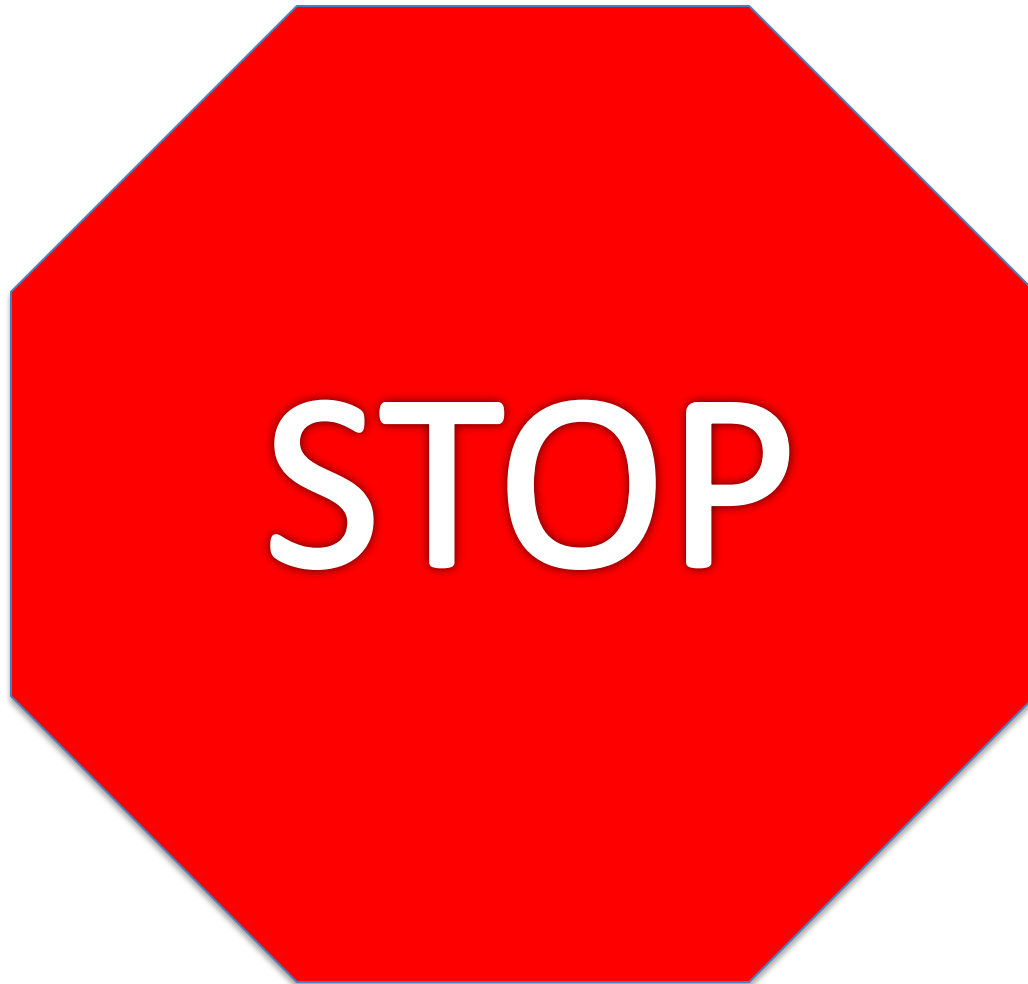
Identifying Proportional Reasoning Problems

Identifying Proportional Reasoning Problems

<p>TYPE of problem:</p> <p>NOTES:</p>	<p>1. Kris and Rich like to skate laps together around an ice rink since they both skate at the same constant rate. Today, Kris started skating first. By the time that Kris had completed 9 laps, Rich had completed 3 laps. How many laps will Kris complete by the time that Rich completes 15 laps?</p>
<p>TYPE of problem:</p> <p>NOTES:</p>	<p>2. If 1 football player weighs 280 pounds, what is the total weight of the 11 starting football players?</p>
<p>TYPE of problem:</p> <p>NOTES:</p>	<p>3. Between them, John and Mark have 32 marbles. John has 3 times as many as Mark. How many marbles does each boy have?</p>
<p>TYPE of problem:</p> <p>NOTES:</p>	<p>4. Three bags of mulch weigh 21 pounds. How many pounds do 8 bags of mulch weigh?</p>

<p>TYPE of problem:</p> <p>NOTES:</p>	<p>5. Mrs. Jones is considering which box of Lucky Charms to buy. There is a 16 oz. box for \$3.36 and a 12 oz. box for \$2.64. Which box of Lucky Charms is the better buy? Explain your reasoning.</p>
<p>TYPE of problem:</p> <p>NOTES:</p>	<p>6. Janie is making a quilt that has a block with two similar triangles, VWX and VYZ. Find the length of side VZ.</p> 
<p>TYPE of problem:</p> <p>NOTES:</p>	<p>7. Alisa is painting her living room. She can paint the entire living room in 4 hours. Assuming that Karen can complete the job in the same amount of time as Alisa, How long will it take Karen and Alisa to paint the living room together?</p>

Hold on a Minute...



Progression of Proportional Reasoning

I. Gauging Additive vs. Multiplicative Thinking

II. Qualitative Reasoning

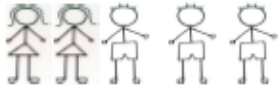
III. Quantitative Reasoning

- Part-Part-Whole
- Associated Sets
- Well-known Measures
- Scalar Growth

How Much, How Many?

How Much vs. How Many

Which family has more girls? Explain your reasoning.

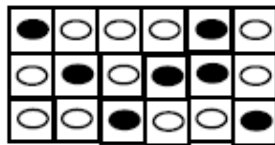
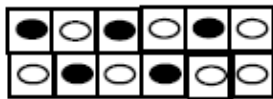


The Jones Family



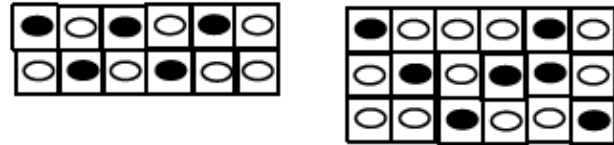
The King Family

Each of the cartons below contains some white eggs and some brown eggs. Which container has more brown eggs? Explain your reasoning.



Classroom Discussion: Bert's solution

Bert solved the problem, "Which container has more brown eggs?"



BERT'S SOLUTION:

I put them into 6 packs like this



Then here, I have $2\frac{1}{2}$ brown eggs in each.

Here, I have $2\frac{1}{3}$ eggs in each.

Is he correct? Explain his thinking.

Cocoa Problem

Name _____

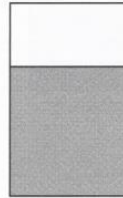
Date _____

Cocoa

1. Thermos A contains cocoa with a stronger chocolate taste. If one scoop of cocoa mix is added to Thermos A and one cup of hot water is added to Thermos B, which thermos contains the cocoa with the stronger chocolate taste? Explain your answer.



Thermos A

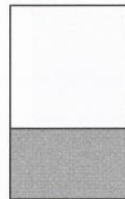


Thermos B

2. Thermos A and Thermos B contain cocoa that tastes the same. If one scoop of cocoa mix is added to both Thermos A and Thermos B, which thermos contains the cocoa with the stronger chocolate taste? Explain your answer.

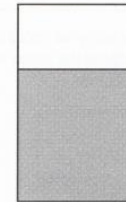


Thermos A

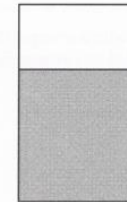


Thermos B

3. Thermos A contains cocoa with a weaker chocolate taste. If one scoop of cocoa mix is added to both Thermos A and Thermos B, which thermos contains the cocoa with the stronger chocolate taste? Explain your answer.

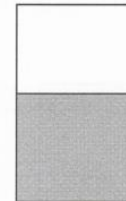


Thermos A

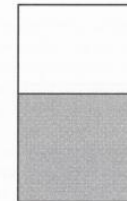


Thermos B

4. Thermos B contains cocoa with a stronger chocolate taste. If one scoop of cocoa mix is added to Thermos A and one cup of hot water is added to Thermos B, which thermos contains the cocoa with the stronger chocolate taste? Explain your answer.



Thermos A



Thermos B

Proportional Reasoning with Cuisenaire Rods

Proportional Reasoning with Cuisenaire Rods

Determine if Set A is equivalent to set B and explain your reasoning.

Set A	Set B	Equivalent yes/no	Reasoning
White is to yellow	Red is to orange		
White is to green	Green is to dark green		
Dark green is to blue	Green is to yellow		
Brown is to orange	Purple is to yellow		
Green is to blue	White is to red		

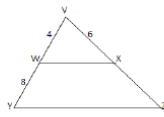
Fill in set B with the color that makes an equivalent relationship with Set A and determine the proportional relationship.

Set A	Set B	Proportional Relationship
White is to purple	Red is to	
Red is to purple	Purple is to	
Yellow is to red	Orange is to	
(orange + red) is to brown	Green is to	

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Part-Part-Whole

Proportional Reasoning with Counters

Part-Part-Whole Proportional Reasoning with Counters

In a class of 27 students, the ratio of girls to boys is 3:6. Use the counters to illustrate your answers to questions #1-3; then draw your solution.

1. Half of the class is girls. True or False
2. How many girls are in the class?
3. How many boys are in the class?
4. Explain how you would find the number of boys and girls in this class without using counters.
5. Keeping the same ratio, how many boys would be in a class of 36 students? Prove your answer with a picture.
6. If there are 30 students in the class, can I have a ratio of girls to boys of 2:5? Explain.
7. Create your own "part-part-whole story problem to share with the class.

Associated Sets & Well-Known Measures

2.1 Mixing Juice

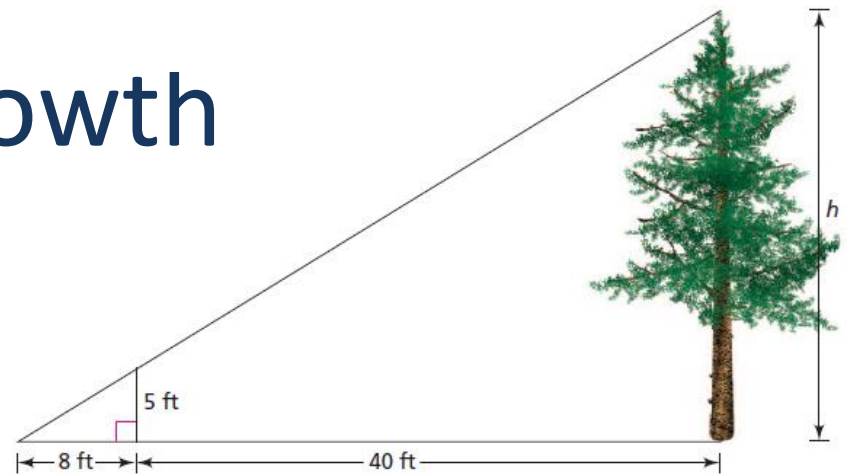
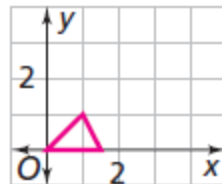
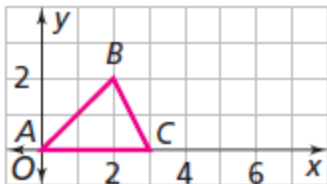
Julia and Mariah attend summer camp. Everyone at the camp helps with the cooking and cleanup at meal times. One morning, Julia and Mariah make orange juice for all the campers. They plan to make the juice by mixing water and frozen orange-juice concentrate. To find the mix that tastes best, they decide to test some mixes.

Mix A	Mix B
2 cups concentrate	5 cups concentrate
3 cups cold water	9 cups cold water

Mix C	Mix D
1 cup concentrate	3 cups concentrate
2 cups cold water	5 cups cold water



Scalar Growth



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RESOURCES

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